SONOBUOY LOCATION

by

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THESIS

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ABSTRACT

In airborne Anti-Submarine Warfare operations there is a critical requirement for maintaining an accurate relative plot of the sonobuoys with respect to the aircraft. This study proposed a method for locating sonobuoys in a pattern using aircraft-to-buoy slant range information. The method did not use triangulation procedures and attempted to minimize the restrictions placed on the aircraft. The study showed the feasibility of the proposed methodology and the approximate errors to be encountered.

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ACKNOWLEDGMENT

Associate Professor Robert R. Read of the Naval Postgraduate School Department of Operations Analysis has been of invaluable assistance in the development of the iterative process for convergence in the Sonobuoy Location portion of this study. His willingness to devote his time to this project not only ensured its timely completion but his knowledge and experience provided an education of rewarding value.

[illegible]

I. INTRODUCTION

The fixed wing aircraft plays an important role in airborne antisubmarine warfare (ASW) today. The primary tactical employment of the aircraft is through the use of active and passive electronic sensors, the most useful of which is the sonobuoy. Sonobuoys are placed by the aircraft in various geometric patterns in the water as the tactical situation predicts. The primary phases in the prosecution of a contact are search, detection, classification, localization and attack. Except in the rare case of a visual sighting of a submarine, the first four phases are almost exclusively conducted by the use of passive and active sonobuoys which transmit data, analyzed by sensor equipment on board the aircraft. This information is then synthesized by the aircraft tactical commander to yield an assessment of the tactical situation.

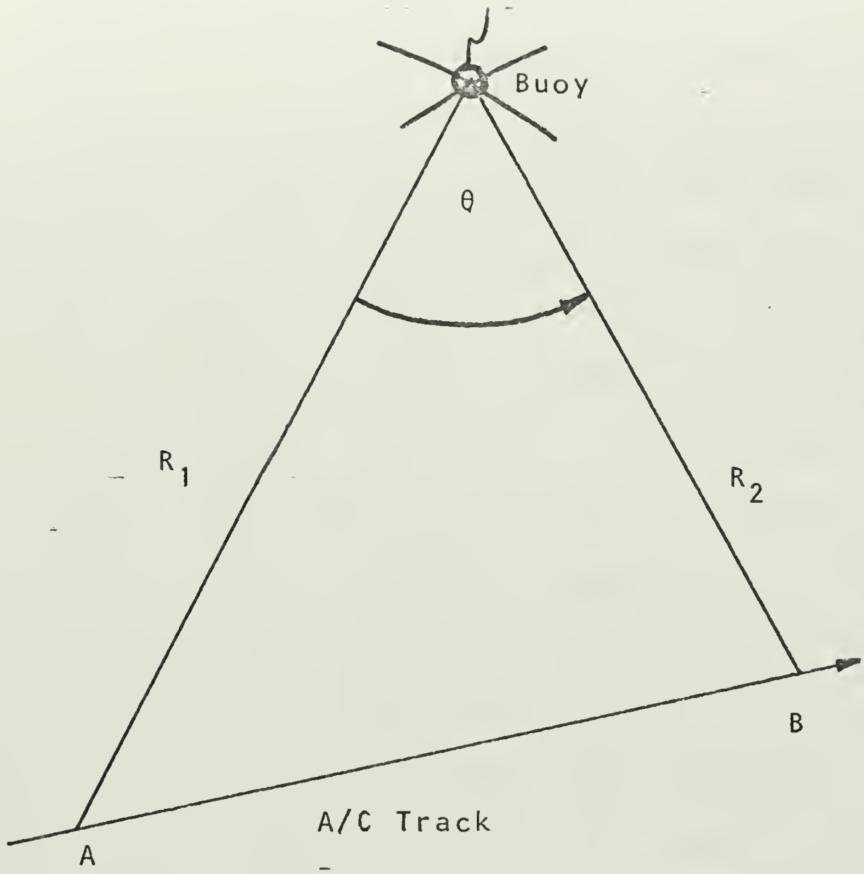
One major problem in airborne ASW is the lack of accurate aircraft/sonobuoy referencing. Surface wind, sea currents and errors in navigation quickly disperse the actual buoy pattern location from its assumed location. Moreover the buoys within the pattern frequently drift in a random manner, particularly when the distance between buoys is great. It is necessary to periodically update the actual buoy positions by a visual or electronic "mark on top." The flexibility of the aircraft is obviously reduced by this requirement. In the fine localization phase where a combination of two or more buoys in close proximity to the target

is providing the aircraft with fix data, it is necessary to fly outbound to the fix after passing over the closest buoy providing data. This tactic is necessary to provide the aircraft with final, accurate buoy location information just prior to the attack. Frequent missed kills are attributed to attacking a fix position generated by buoys whose accurate locations with respect to the aircraft were not known or recently updated.

A better method for determining the location of sonobuoys is required. A method for locating buoy coordinates relative to the aircraft, remote from the buoy, would eliminate the need for "on top" repositioning. Such a method would also offer the additional aircraft flexibility required for the eventual increase in the tactical load placed on the crew.

A system under development [Ref. 1,2] will provide for a slant range measurement to the buoy. This slant range would be more accurate than the aircraft navigation system and could be converted to horizontal range knowing the aircraft altitude. The range to a buoy can be taken at any time after it has been placed in the water up to the life of the buoy and in any tactical environment providing the aircraft is at sufficient altitude to receive the signal. The use of the ranges provided by the buoys under the proposed system [Ref. 1] serves as the subject of this study.

While triangulation methods obviously provide a solution to the buoy location problem [Fig. 1] th aircraft



R_i = horizontal range from A/C to buoy

A = A/C location on track when first range measured

B = A/C location on track when second range measured

θ = separation angle between range measurements

FIGURE 1

STANDARD TRIANGULATION PROCEDURE

track, line AB, between the buoy rangings required to complete triangulation should be approximately equal to the range of the buoy. The buoy fix position is determined by the intersection of the range arcs to the buoy from the aircraft position. The errors realized in the buoy fix position are, for the most part, due to inaccuracies in the ranging system and inaccuracies in navigation from points A to B. If the leg AB [Fig. 1] representing the aircraft track is allowed to be in the order of length equal to the range to the buoy such that the range arcs intersect as close to perpendicular as possible, then the errors due to navigation and range measurement would be transferred to the buoy fix error in a nearly one-to-one ratio. If, however, the aircraft track, leg AB, between range measurements is short, inferring an angle θ of only a few degrees, then the range arcs will intersect nearly parallel. The bearing error of the resulting buoy fix position may be extremely large.

In that the length of the aircraft track required for optimal error reduction in triangulation necessarily restricts the flexibility of the aircraft, this study will focus on a method of minimizing the error by statistical methods using a rapid series of rangings to the buoy over a short interval. It is felt that the statistical methods of error reduction will allow an improved solution to the problem over standard triangulation.

II. NATURE OF THE PROBLEM

The major portion of this study is centered around (1) determining a method for locating sonobuoys relative to the aircraft, (2) feasibility of the system with respect to the deployment of the aircraft in a tactical environment, and (3) the errors anticipated and how sensitive the procedure is to these errors. A residual effort conducted in conjunction with the development of the sonobuoy location problem provides a conceptual solution method for determining an accurate aircraft ground speed and true course. The development concepts for the ground speed and true course determination are not complicated and should therefore be adapted easily to any aircraft in the fleet today which would carry the associated sonobuoy ranging equipment.

Range information from the buoys will be available continuously, sampled as desired. The slant range will be converted to horizontal range as described by Ellis [Ref. 1, Fig. 4]. There is no inherent ambiguity problem such as mirror images of fix positions and the solution is not quadrant or half plane restricted. Minor restrictions are placed on the aircraft for short periods of time. The aircraft, in most cases, will be required to fly straight and level at a constant true airspeed while sampling ranges to the buoys; however these intervals are quite short, four to twenty seconds as the tactical situation dictates. This restriction is not considered to be excessive as will be explained in the formal development of techniques for

sonobuoy location and ground speed determination. In certain cases the measured ranges to the sonobuoys will be the only input required by the system, which generates a very appealing aspect to the solution technique. In other cases the aircraft coordinates as well as the measured range information will be required as the only inputs.

It should be noted here that navigational errors producing erroneous geographical track of the aircraft are not critical in the problem solution as such. The primary problem being attacked by this study is that of locating the sonobuoy relative to the aircraft. If, in fact, the DR track is erroneous then the positions determined for the buoys will be erroneous; however, their positions relative to the aircraft will be accurate as they will be plotted relative to the aircraft DR position. Accurate geographical track and sonobuoy location information can be updated subsequent to the attack phase of the problem. Experience has shown that two aircraft in the process of relieving each other may have a relative error of up to 15 miles in geographical DR tracks. Therefore the transfer of geographical positions of buoys within a pattern is frequently unsatisfactory. When it is necessary to turn the problem over to a relieving aircraft, it will be easier to transfer a relative picture, that is, range and bearing of all other buoys in the pattern from one reference buoy close to the relieving aircraft.

Although the P3-C aircraft will far surpass any existing

ASW airborne weapon system in the fleet today it is not anticipated that the models used currently will be phased out in the near future. It is important to consider developing a system which may, with minimal modification, be adapted to current models of ASW aircraft. The ground speed and true course determination is developed with this in mind. Considering the contribution of locating sonobuoys from remote positions to the prosecution of a contact, it may be well worth while to consider the installation of a limited purpose computer in the current model VP aircraft vice more costly alternatives.

III. BUOY RANGING CONCEPT

The sonobuoy has a transmitter built in for the purpose of transmitting to the aircraft the sensor signals received from the sonobuoy hydrophones. The carrier frequency associated with this signal is channelized in each buoy for buoy discrimination. A buoy ranging signal described in detail by Ellis [Ref. 1] will be transmitted over the existing aircraft receivers. No new receivers or transmitters for the signal will be required. A modification to the detector section of the sonobuoy receiver will be required for signal separation.

The sonobuoy ranging signal is generated by a crystal oscillator placed in the buoy. The method of range determination is by comparison of the phase of the buoy signal with that of a standard frequency generated on board the aircraft. The phase difference is converted to slant range.

The sonobuoy undergoes a warmup period in the aircraft prior to drop. It generates a stable frequency, f_r, and is compard with the aircraft frequency, f_r'. The phase difference at this time and at zero distance is measured, $\Delta\theta_o$. At any time, t, after the sonobuoy is dropped in the water a new comparison is made between f_r and f_r', and the phase $\Delta\theta_t$, is determined. The resulting phase difference, $\Delta\theta_r$, where $\Delta\theta_r = \Delta\theta_t - \Delta\theta_o$, is then converted to slant range; subsequently to horizontal range. The buoy range frequency has a wavelength of approximately 75 kilometers. One cycle difference in phase of the buoy signal from the reference

is equivalent to 75 km. distance. A buoy at a range of 10 km. and 85 km. would yield the same phase difference causing an ambiguity. It must be known what multiple of 75 km. the buoy is from the aircraft. It is not anticipated that this ambiguity will cause erroneous ranging.

The range errors generated by the system are covered in detail by Ellis [Ref. 1] but one error of primary concern in the tactical employment of the system is the linear increase in frequency with respect to time within the buoy oscillator. Complete frequency stabilization is not yet a "state-of-the-art" realization. Testing to date with the oscillator currently in use in the buoy has generated an average drift rate comparable to a range error of 1040 meters per hour. If a means was devised for the subtraction of the linear drift rate as a function of time, the remaining errors in the system could be aggregated to a classification of normal with a mean of zero and a variance of σ_ε^2. This aggregation of errors being $N(0,\sigma_\varepsilon^2)$ would allow a broad application of statistical methods for the solution of the error reduction problem in the tactical employment of the buoy range. It is therefore assumed that in the following study the average linear error rate is eliminated from the problem and the assumption that the remaining errors are $N(0,\sigma_\varepsilon^2)$ will motivate the development.

IV. SUMMARY OF PROBLEM SOLUTION TECHNIQUES

The following methods are developed for the solution of (1) Sonobuoy Location, (2) Aircraft Ground Speed and (3) Aircraft True Course. This section provides a non-technical summary of the problem geometry, basic solution approach and the assumptions and restrictions governing the technique used in each of the three problem areas mentioned above. It is designed as a background preface to the more detailed mathematical development and should by itself acquaint the reader with a complete broad picture of what has been accomplished. The mathematical development will be included in a subsequent section.

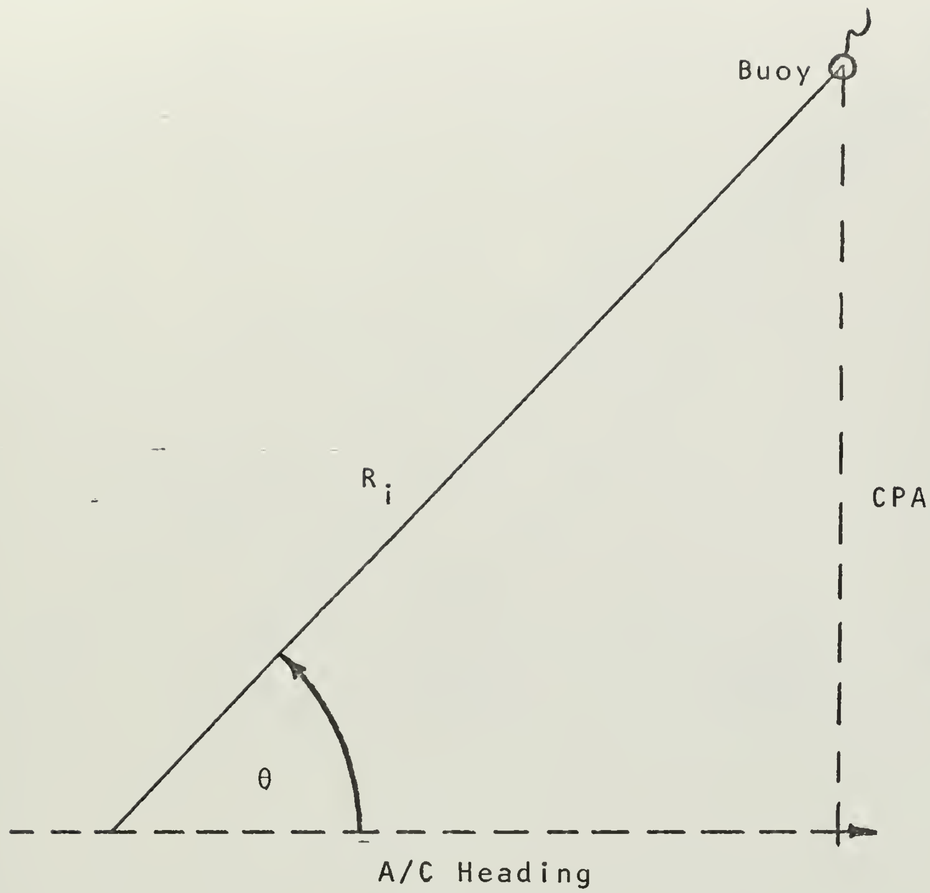
A. SONOBUOY LOCATION

It is desired to provide a system whereby the aircraft may, in any phase of the tactical prosecution of a contact, locate any or all of the sonobuoys in the pattern with respect to the aircraft. To provide the aircraft with the flexibility needed in the more acute phases of the problem, the system must not place heavy restrictions on the maneuverability of the aircraft such as is required in standard triangulation techniques. A method whereby the aircraft could obtain locations from a single position and time or with a minimum advance leg would certainly be preferred.

Recalling the discussion in the last paragraph of the introduction, an extended aircraft track, line AB [Fig. 1], would have to be flown in order that θ be sufficiently large to reduce fix error of the buoy position. In a tactical

situation it may not be feasible for the aircraft to fly the leg AB. If the aircraft desired to maneuver between points A and B it must do so at the possible cost of additional navigational errors as aircraft navigation becomes degraded in many cases as a result of maneuvering.

The geometry in Figure 2 was considered as a starting point for the development for sonobuoy location. This basic geometry will prevail in the description of the problem. Consider an aircraft with a buoy in the relative position shown. The aircraft is required to hold any true heading and airspeed for as long as it is required to measure ranges R_1, R_2 and R_3 [Fig. 3]. The time interval between each range measurement is short, approximately five seconds. At 200 kts. an aircraft travels 111 yds./second. As many ranges as desired may be measured but more time is required and the additional accuracy obtained by more ranging is not significant as a trade off in a critical tactical situation. This will be a tactical decision. The study uses three range measurements throughout the descriptive phase. Additional ranges were used in testing. It is assumed that in the short period of time the aircraft is required to fly straight and at a constant air speed that the coordinates (A_i, B_i) [Fig. 3] are accurate relative to each other. Any inaccuracies occuring will be absorbed in final fix error. The ranges R_i and the aircraft coordinates (A_i, B_i) are inputs and are assumed to be known. The buoy coordinates (X, Y) are unknown.



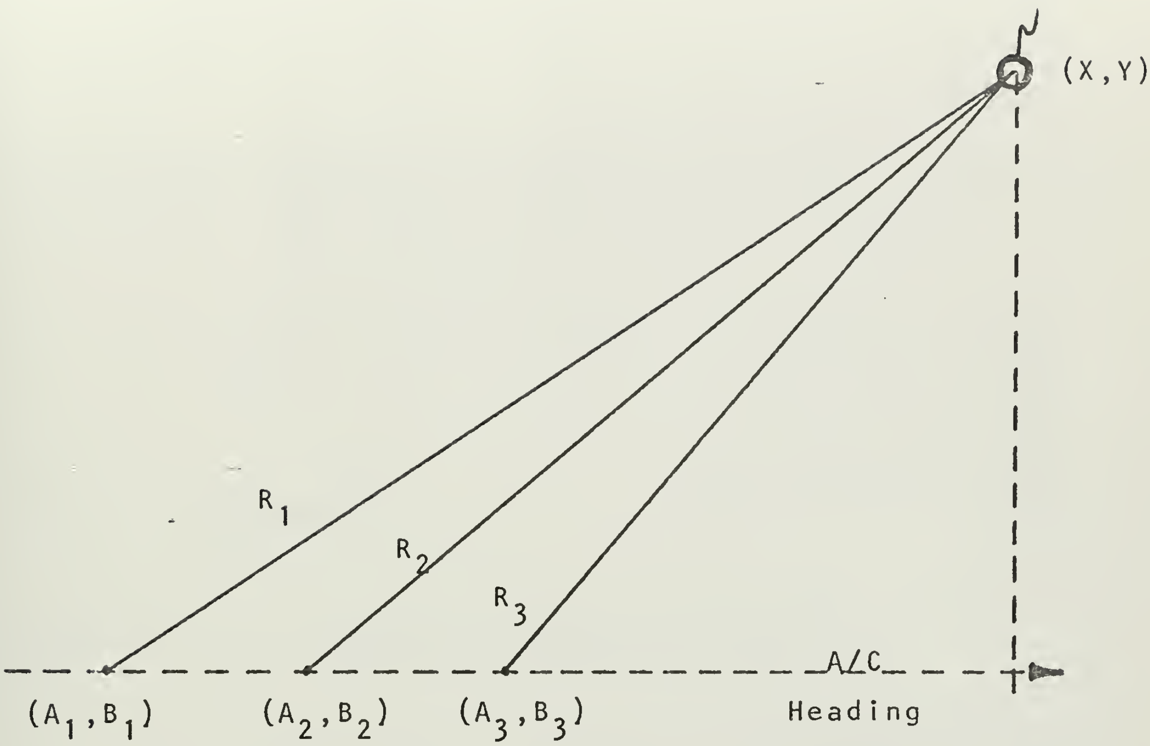
R_i = horizontal range from A/C to buoy

CPA = closest point of approach to buoy with respect to the A/C

θ = relative bearing of buoy with respect to the A/C

FIGURE 2

BASIC TWO DIMENSIONAL PROBLEM GEOMETRY



R_i = horizontal range to buoy as measured by A/C $i=1,2,\ldots,n$

(A_i,B_i) = A/C coordinates at time of measurement of range R_i to buoy $i=1,2,\ldots,n$

(X,Y) = coordinated of buoy (unknown)

FIGURE 3

SONOBUOY LOCATION PROBLEM GEOMETRY

The model used to determine the (X,Y) position of the buoy is the method of Least Squares. The basic assumption for the use of this model is that the errors are $N(0,\sigma_{\varepsilon}^{2})$. The approach is to minimize the sum of the squares of the errors between the measured ranges to the buoy and the actual ranges. The equation for the Sum of Squares;

$$SS = \sum_{i=1}^{n} (O_i - E_i)^2 \qquad \text{where;} \quad O_i\text{=observed value}$$

$$E_i\text{=expected value}$$

$$= \sum_{i=1}^{n} (\varepsilon_i)^2 \qquad O_i - E_i = \varepsilon_i \text{ (error term)}$$

is adapted to the sonobuoy ranging problem;

$$SS = \sum_{i=1}^{n} (R_i - R_i')^2 \qquad \text{where;} \quad R_i\text{=measured range}$$

$$R_i'\text{=expected value}$$

$$=[(X-A_i)^2 + (Y-B_i)^2]^{\frac{1}{2}}$$

$$R_i - R_i' = \varepsilon_i$$

The approach of minimizing the sum of the error squared terms is that of evaluating the first derivative of SS at zero with respect to the X and Y values desired;

$$SS = \sum_{i=1}^{n} (\varepsilon_i)^2 = \sum_{i=1}^{n} \{R_i - [(X-A_i)^2 + (Y-B_i)^2]^{\frac{1}{2}}\}^2$$

$$\frac{\partial SS}{\partial X} = \frac{\partial SS}{\partial Y} = 0.$$

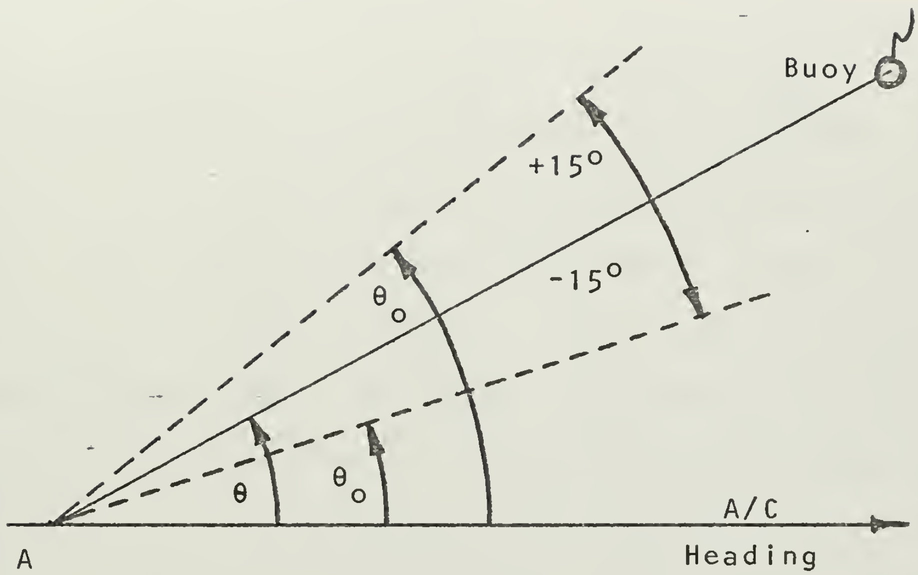
This development produces a set of normal equations from which the values of X and Y are determined by an iterative

process.

The iterative process requires that an estimate of the buoy angle relative to the aircraft, θ, be supplied. This initial angle, θ_0, provides a starting point for the process which by making alternately successive approximations of X, Y and θ the process converges on the values of X and Y with the minimum error possible for the geometry. Values of θ_0 up to and including $\pm 15^{\circ}$ in error of the actual angle θ were tried [Fig. 4]. There was little effect on the rapid convergence of X and Y to within acceptable values. In consideration of the latitude in choosing a value of θ_0 it is felt that the bearing supplied to the buoy by the DME indicator is satisfactory as an input value. The value of θ_0 was considered to have been chosen as an estimate of the θ for the mid-range measurement.

Two studies were conducted to determine the feasibility of the Least Squares model for the solution of the sonobuoy location problem. The first study was a sensitivity analysis to determine the effect of the following on the convergence rate of the process; (1) distance to the buoy, (2) relative angle of the buoy, θ, (3) values of θ_0 as an estimate of and (4) number of ranges used in the solution of the problem.

The second study was an error analysis to examine the effect of range error on the final values of buoy coordinates, X and Y. Errors of varying magnitudes were applied to the ranges in different combinations at different relative



A = A/C location when horizontal range to buoy measured.

θ = actual relative angle of buoy with respect to A/C heading

θ_o = estimated values of relative angle of buoy; error applied in 1° increments up to ± 15° of actual relative angle

FIGURE 4

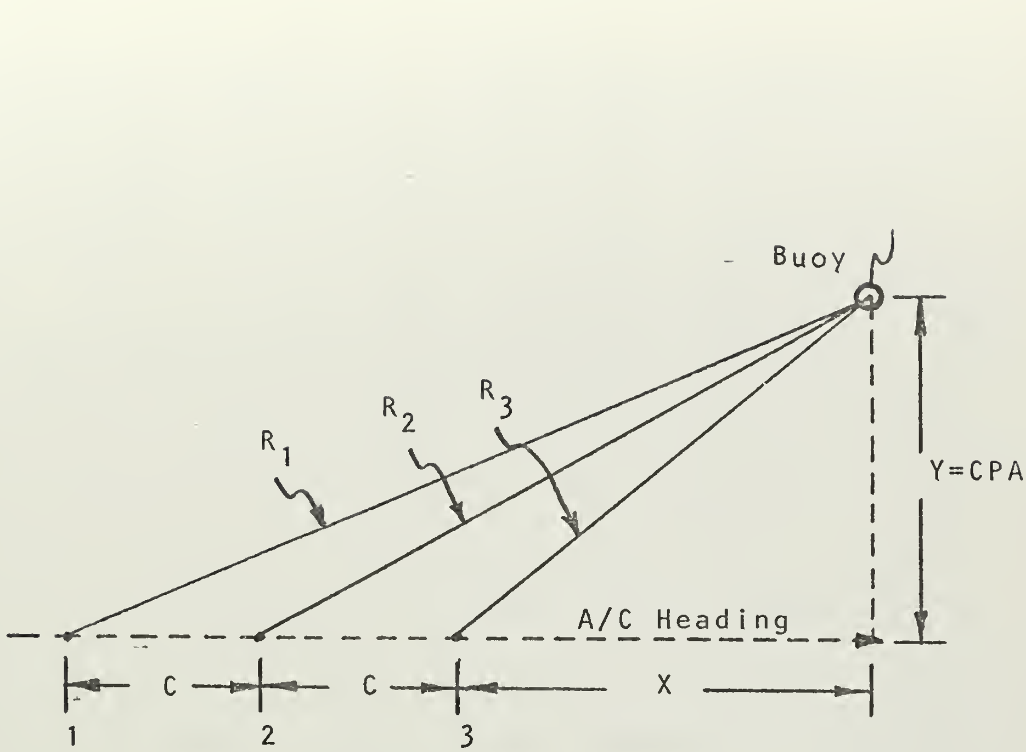
RELATIONSHIP BETWEEN ESTIMATED AND ACTUAL VALUES OF RELATIVE BUOY ANGLE WITH RESPECT TO A/C HEADING

buoy angles. The results of these studies are explained in detail in Appendix A.

B. GROUND SPEED DETERMINATION

The development of the method of ground speed determination is based on the concept that if an aircraft was approaching a buoy (θ^{o}relative bearing) which was providing accurate range information then the change in the horizontal range to the buoy with respect to time would be, exactly, the ground speed. This is a fast, accurate and simple way to determine ground speed if it could be ascertained that the relative bearing was exactly θ^{o} relative. This is difficult to do as the DME indicator does not provide that accuracy. As the relative angle of the buoy increases from zero the first derivative of the range to the buoy with time becomes a function of the cosine of the relative angle. If there existed a scheme for determining the change in range to the buoy with respect to time with the relationship of the relative angle then the ground speed could be determined for any relative angle for the buoy.

In Figure 5 a geometry similar to that of the previous section is used. Consider an aircraft approaching a buoy as indicated. Were the aircraft to continue on this heading it would pass the buoy at its closest point of approach (CPA) at a point perpendicular to the aircraft track. In the problem solution the aircraft chooses any buoy whose relative bearing is close to the aircraft heading. The aircraft is then required to fly straight and level at a



CPA = closest point of approach to buoy with respect to the A/C heading

R_i = horizontal ranges to the buoy measured at A/C locations 1, 2 and 3

C = equal distances (assumed) generated by flying equal times at a constant true airspeed between points 1, 2 and 3

Note: Buoy coordinates are not known, moreover are not required for this problem.

FIGURE 5

GROUND SPEED DETERMINATION PROBLEM GEOMETRY

constant true air speed for a time interval sufficient to measure from three to eight ranges to the buoy in question. The time between each measurement is short, two to four seconds. The time between each range, C, should be exactly equal. This could be done by electronic time gating. This time, C, also represents equal distance between range measurements. It is assumed that for short periods of time and by electronic time gating, these distances will be equal. Any errors are absorbed in the total ground speed error.

The actual or estimated position of the buoy is not required as an input to the problem. Also it is not required to know the heading or airspeed of the aircraft, only that they are constant. The evaluation of C determines the ground speed. The use of the Pythagorean theorem produces three equations for the three ranges measured;

$$(X+2C)^2 + Y^2 = R_1^2 \quad \text{for range measured at 1}$$
$$(X+C)^2 + Y^2 = R_2^2 \quad \text{for range measured at 2}$$
$$X^2 + Y^2 = R_3^2 \quad \text{for range measured at 3}$$

The magnitude of the ranges only are known and not the direction. There are three equations and three unknowns; X, Y and C. The values for C are evaluated from the above equations for the three ranges;

$$C = \sqrt{\frac{R_1^2 - 2R_2^2 + R_3^2}{2}}$$

By a similar development C is determined for any number of

ranges measured. Up to eight ranges are used in the detailed mathematical development included later.

Two studies were conducted to determine the feasibility of the ground speed determination method. The first study was a sensitivity analysis to determine the effect on ground speed error by varying the following; (1) number of ranges used, (2) time between range measurement, (3) ranges to the buoy and (4) the relative bearing of the buoy.

The second study was an error analysis on the ground speed as affected by errors in the measured ranges. The results of these studies are combined in Appendix B.

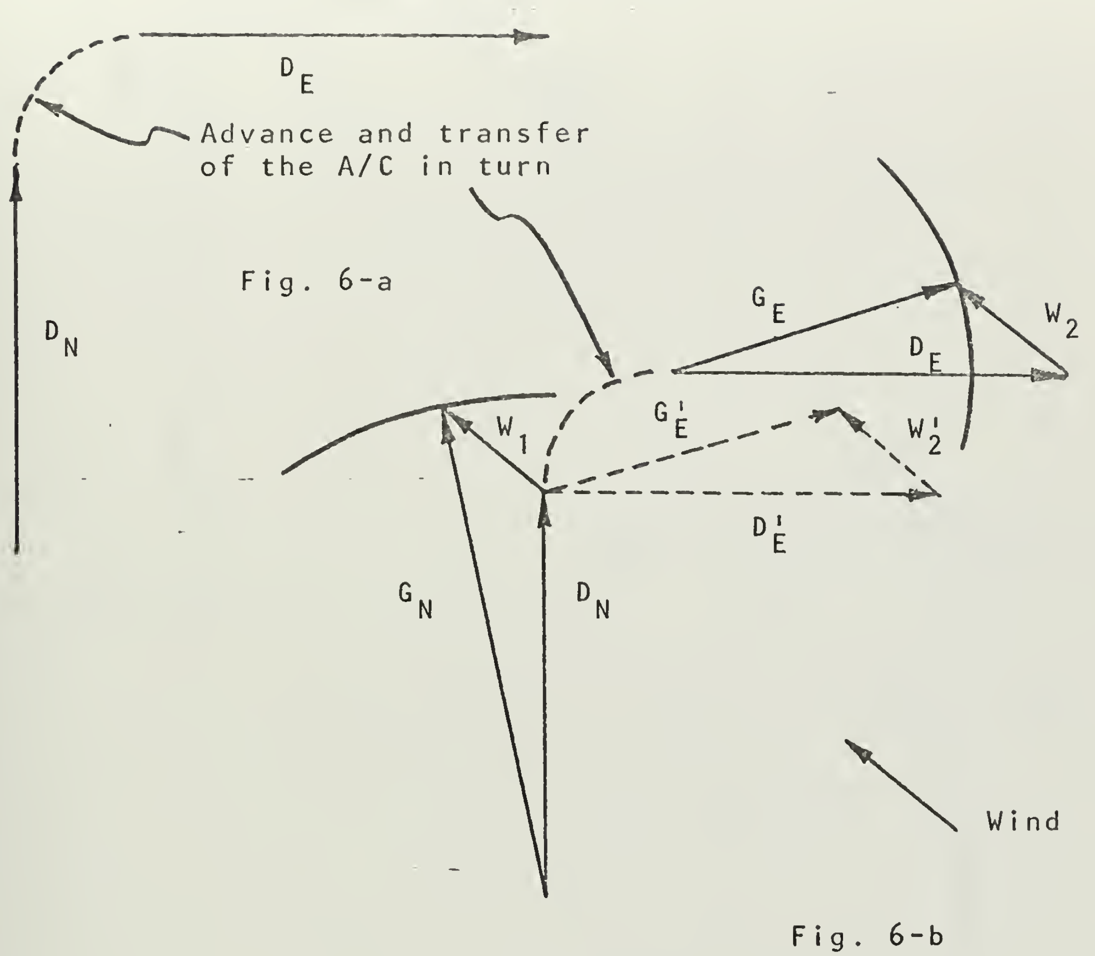
This method for determining ground speed is expected to have limited application, however, because of the simplicity of concept it should be quite reliable to supplement breakdown of more sophisticated inertial and dopplar navigational systems. Frequently, in fine localization, a timed run out from some reference to a fix position for attack is required. It is felt that this ground speed determination method can be utilized on the final leg to more accurately determine the ground speed and hence a more accurate time to fix on that leg. The aircraft will maintain a true course and airspeed to that fix position which meets the requirements for the solution.

C. TRUE COURSE DETERMINATION

It was felt that the use of the range measurement to a buoy as the only input could provide additional information, if applied properly, in evaluating a "drift vector". Consider

in Figure 6-a, an aircraft flying a true heading of due North. This true heading is held, at a constant true air speed (TAS), for a time t_N. Vector D_N represents the true air speed times the time flown, t_N. The aircraft then turns to a true heading of East and holds this heading, at the same TAS, for a time t_E where $t_E = t_N$. While on each of these headings the aircraft determines ground speed by methods described in the previous section. A different buoy is used for ground speed determination for each of the two headings. If a wind from the Southeast were affecting flight, the ground speed for the northerly leg would be greater than the TAS and the ground speed for the easterly leg would be less than the TAS, [Fig. 6-b]. The triangle $G_E'W_2'D_E'$ shows the second leg geometry without the advance and transfer of the aircraft in the turn. The magnitude only and not the direction of these ground speed vectors is known. It should be noted that any initial true heading and TAS could be flown and a subsequent right or left turn could be executed. Maximum accuracy of the drift vector is obtained when the subsequent heading is perpendicular to the initial heading.

In the development for the determination of a drift vector, the advance and transfer of the aircraft was eliminated in further diagrams and does not alter the geometric accuracy of the development. The actual path of the aircraft, less the advance and transfer, [Fig. 7-a], is represented by the vectors G_N and G_E whereas the vectors D_N and



$$\left.\begin{aligned} D_N &= t_N \cdot TAS \\ D_E &= t_E \cdot TAS \end{aligned}\right\} \text{ where } t_N = t_E \quad D_N = D_E$$

$$\left.\begin{aligned} W_1 &= t_N \cdot (\text{Windspeed + additional errors}) \\ W_2 &= t_E \cdot (\text{Windspeed + additional errors}) \end{aligned}\right\} W_1 = W_2$$

$G_N = t_N \cdot$ (Groundspeed for North leg) magnitude only

$G_E = t_E \cdot$ (Groundspeed for East leg) magnitude only

FIGURE 6

TRUE COURSE DETERMINATION PROBLEM GEOMETRY

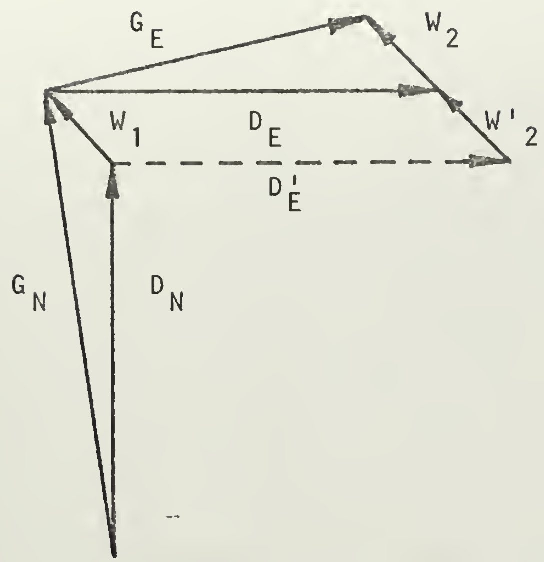


Fig. 7-a

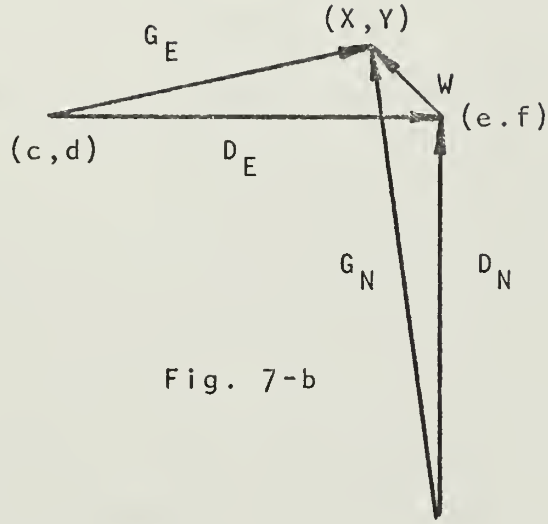


Fig. 7-b

W = drift vector = wind vector plus additional error components

Known Values:
A/C assumed position coordinates, (a,b), (c,d) and (e,f)
vectors D_N and D_E, both magnitude and direction
vectors G_N and G_E, magnitude only

Unknown Values:
vectors G_N and G_E, direction only
coordinates (X,Y)
drift vector, W, both magnitude and direction

FIGURE 7

TRUE COURSE DETERMINATION SOLUTION GEOMETRY

D_E' represent the "no wind" track. The geometry of the problem was altered for the purpose of the mathematical solution technique. The triangle $G_E W_2 D_E$ was shifted [Fig. 7-b] such that W_2 coincided with W_1 since $\bar{W}_2 = \bar{W}_1$. The vector D_E did not change in orientation thus the relationship between the two triangles, $G_E W_2 D_E$ and $G_N W_1 D_N$, was not changed. The resulting form revealed that the vectors G_N and G_E had the same terminal point. The known and unknown values in the figure are listed at the bottom of Figure 7.

A pair of quadratic equations were used to solve for the terminal point, (X,Y);

$$(X-a)^2 + (Y-b)^2 = G_N^2$$

$$(X-c)^2 + (Y-d)^2 = G_E^2$$

The magnitude of the drift vector, W, was then solved, as with the determination of the coordinates (X, Y) both the coordinates of its origin and terminal point were known.

$$W = \sqrt{(X-e)^2 + (Y-f)^2}$$

The resulting drift vector was then used in conjunction with TAS and true heading to determine an actual course and ground speed.

V. SONOBUOY LOCATION TECHNIQUES

From the description of the basic sum of squares model in section IV, the sum of squares equation for the particular geometry in Figure 3 was obtained;

$$SS = \frac{1}{n} \sum_{i=1}^{n} \{R_i - [(X-A_i)^2 + (Y-B_i)^2]^{\frac{1}{2}}\}^2$$

In further development let

$$x_i = X-A_i \tag{1}$$

$$y_i = Y-B_i \tag{2}$$

then

$$SS = \frac{1}{n} \sum_{i=1}^{n} [R_i - (x_i^2+y_i^2)^{\frac{1}{2}}]^2 \tag{3}$$

Equation (3) was then differentiated with respect to X and Y and evaluated at zero, i.e., $\frac{\partial SS}{\partial X} = \frac{\partial SS}{\partial Y} = 0$

$$\frac{\partial SS}{\partial X} = \frac{2}{n} \sum_{i=1}^{n} [R_i - (x_i^2+y_i^2)^{\frac{1}{2}}](-\tfrac{1}{2})(2x_i)(x_i^2+y_i^2)^{-\frac{1}{2}} = 0$$

$$\frac{1}{n} \sum_{i=1}^{n} \frac{R_i x_i}{(x_i^2+y_i^2)^{\frac{1}{2}}} - \frac{1}{n} \sum_{i=1}^{n} x_i = 0 \tag{4}$$

Let (a) be

$$\frac{1}{n} \sum_{i=1}^{n} \frac{R_i x_i}{(x_i^2+y_i^2)^{\frac{1}{2}}}$$

and let (b) be

$$\frac{1}{n}\sum_{i=1}^{n} x_i$$

Each term of the expression (a) was multiplied, both numerator and denominator, by $\frac{1}{x_i}$

$$\frac{1}{n}\sum_{i=1}^{n}\frac{R_i x_i (\frac{1}{x_i})}{(\frac{1}{x_i})(x_i^2+y_i^2)^{\frac{1}{2}}} = \frac{1}{n}\sum_{i=1}^{n}\frac{R_i}{(1+\frac{y_i^2}{x_i^2})^{\frac{1}{2}}} = \frac{1}{n}\sum_{i=1}^{n}\frac{R_i}{(1+\tan^2\theta_i)^{\frac{1}{2}}}$$

where θ_i is the relative angle of the buoy at the time the range R_i was measured, and

$$\tan\theta_i = \frac{y_i}{x_i} = \frac{Y-B_i}{X-A_i} \tag{5}$$

Thus (a) becomes;

$$\frac{1}{n}\sum_{i=1}^{n}\frac{R_i}{(\mathrm{Sec}^2\theta_i)^{\frac{1}{2}}} = \frac{1}{n}\sum_{i=1}^{n} R_i\cos\theta_i$$

The second term, (b) was expanded;

$$\frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{n}\sum_{i=1}^{n}(X-A_i) = \frac{X}{n}\sum_{i=1}^{n} - \frac{1}{n}\sum_{i=1}^{n} A_i = X - \frac{1}{n}\sum_{i=1}^{n} A_i$$

The two terms (a) and (b), were then combined;

$$\frac{1}{n}\sum_{i=1}^{n} R_i\cos\theta_i - X + \frac{1}{n}\sum_{i=1}^{n} A_i = 0$$

$$X = \frac{1}{n}\sum_{i=1}^{n} R_i\cos\theta_i - \frac{1}{n}\sum_{i=1}^{n} A_i \tag{6}$$

A similar development for $\frac{\partial SS}{\partial Y} = 0$ yielded

$$Y = \frac{1}{n} \sum_{i=1}^{n} R_i \sin\theta_i - \frac{1}{n} \sum_{i=1}^{n} B_i \qquad (7)$$

The unknowns X and Y could not be solved directly as θ_i is a function of X and Y (see equation 5); therefore, an initial value common for all the θ_i was estimated, namely θ_0, and initial values for X and Y, i.e., X_0 and Y_0 were determined using θ_0 for each θ_i in equations 6 and 7. These values of X_0 and Y_0 in turn were used to update the values for θ_i, namely;

$$\tan\theta_i = \frac{Y_0 - B_i}{X_0 - A_i} \qquad \forall_i \qquad i = 1,2,\ldots,n$$

These values for the θ_i were then substituted in the equations 6 and 7 to determine X_1 and Y_1. This process was repeated until the convergence of X and Y to within acceptable values was attained.

The foregoing iterative process proved to be lengthy. The angles θ_i were quite insensitive to small changes in the positions of X and Y through the iterative process. It was felt that an application of Newton's method in two variables might provide a more rapid convergence. The method involved the expansion of the functions (as determined by equations 6 and 7 using θ_0) in Taylor's Series in two variables about X_0 and Y_0. The expansion would terminate after the linear terms and the remainder ignored. The

The resulting equations would be solved for X and Y and the solutions used for an updated expansion. This process would be repeated until the values of X and Y were within acceptable limits.

The sine and cosine of the relative angles θ_i as appearing in equations 6 and 7 were developed in terms of x_i and y_i (see equations 1 and 2);

$$\cos\theta_i = \frac{1}{(\sec^2\theta_i)^{\frac{1}{2}}} = \frac{1}{(1+\tan^2\theta_i)^{\frac{1}{2}}} = \frac{1}{(1+y_i^2/x_i^2)^{\frac{1}{2}}} =$$

$$\frac{x_i}{(x_i^2+y_i^2)^{\frac{1}{2}}}$$

$$\sin\theta_i = \frac{1}{(\csc^2\theta_i)^{\frac{1}{2}}} = \frac{1}{(1+\cot^2\theta_i)^{\frac{1}{2}}} = \frac{1}{(1+x_i^2/y_i^2)^{\frac{1}{2}}} =$$

$$\frac{y_i}{(x_i^2+y_i^2)^{\frac{1}{2}}}$$

The equations 6 and 7 were then cast into the homogeneous forms below;

$$g_1(X,Y) = X - \frac{1}{n}\sum_{i=1}^{n}\{R_i\frac{x_i}{(x_i^2+y_i^2)^{\frac{1}{2}}} - A_i\} = 0$$

$$g_2(X,Y) = Y - \frac{1}{n}\sum_{i=1}^{n}\{R_i\frac{y_i}{(x_i^2+y_i^2)^{\frac{1}{2}}} - B_i\} = 0$$

The form of the Taylor expansion is

$$g(X,Y)=g(X_0,Y_0)+g_x(X_0,Y_0)(X-X_0)+g_y(X_0,Y_0)(Y-Y_0)+\text{Remainder}$$

Specifically, g_1 expanded;

$$g_1(X,Y)=X_0-\frac{1}{n}\sum_{i=1}^{n}\{R_i\frac{x_i}{(x_i^2+y_i^2)^{\frac{1}{2}}}-A_i\}+[1-\frac{1}{n}\sum_{i=1}^{n}R_i\frac{y_i^2}{(x_i^2+y_i^2)^{3/2}}](X-X_0)$$

$$+[\frac{1}{n}\sum_{i=1}^{n}R_i\frac{x_iy_i}{(x_i^2+y_i^2)^{3/2}}](Y-Y_0) = 0 \qquad (8)$$

and g_2 expanded;

$$g_2(X,Y)=Y_0-\frac{1}{n}\sum_{i=1}^{n}\{R_i\frac{y_i}{(x_i^2+y_i^2)^{\frac{1}{2}}}-B_i\}+[\frac{1}{n}\sum_{i=1}^{n}R_i\frac{x_iy_i}{(x_i^2+y_i^2)^{3/2}}](X-X_0)$$

$$+[1-\frac{1}{n}\sum_{i=1}^{n}R_i\frac{x_i^2}{(x_i^2+y_i^2)^{3/2}}](Y-Y_0) = 0 \qquad (9)$$

with x_i and y_i evaluated at X_0, Y_0.

Equating (8) and (9) to zero yielded the following system of linear equations;

$$[1-\frac{1}{n}\sum_{i=1}^{n}R_i\frac{y_i^2}{(x_i^2+y_i^2)^{3/2}}]X + [\frac{1}{n}\sum_{i=1}^{n}R_i\frac{x_iy_i}{(x_i^2+y_i^2)^{3/2}}]Y$$

$$=\frac{Y_0}{n}\sum_{i=1}^{n}R_i\frac{x_iy_i}{(x_i^2+y_i^2)^{3/2}} - \frac{X_0}{n}\sum_{i=1}^{n}R_i\frac{y_i^2}{(x_i^2+y_i^2)^{3/2}}$$

$$+\frac{1}{n}\sum_{i=1}^{n}\frac{x_i}{(x_i^2+y_i^2)^{\frac{1}{2}}} + \frac{1}{n}\sum_{i=1}^{n}A_i$$

$$\left[\frac{1}{n}\sum_{i=1}^{n} R_i \frac{x_i y_i}{(x_i^2+y_i^2)^{3/2}}\right] X + \left[1-\frac{1}{n}\sum_{i=1}^{n} R_i \frac{x_i^2}{(x_i^2+y_i^2)^{3/2}}\right] Y$$

$$= \frac{X_0}{n}\sum_{i=1}^{n} R_i \frac{x_i y_i}{(x_i^2+Y_i^2)^{3/2}} - \frac{Y_0}{n}\sum_{i=1}^{n} \frac{x_i^2}{(x_i^2+y_i^2)^{3/2}}$$

$$+ \frac{1}{n}\sum_{i-1}^{n} \frac{y_i}{(x_i^2+y_i^2)^{\frac{1}{2}}} + \frac{1}{n}\sum_{i=1}^{n} B_i$$

Cramer's rule was applied to solve for X and Y. The coefficient matrix, H, of X and Y,

$$H = \begin{bmatrix} D_1 & D_3 \\ & \\ D_2 & D_4 \end{bmatrix} = \begin{bmatrix} 1 = \frac{1}{n}\sum_{i=1}^{n} R_i \frac{y_i^2}{(x_i^2+y_i^2)^{3/2}} & \frac{1}{n}\sum_{i=1}^{n} R_i \frac{x_i y_i}{(x_i^2+y_i^2)^{3/2}} \\ & \\ \frac{1}{n} - \sum_{i=1}^{n} R_i \frac{x_i y_i}{(x_i^2+y_i^2)^{3/2}} & 1-\frac{1}{n}\sum_{i=1}^{n} R_i \frac{x_i^2}{(x_i^2+y_i^2)^{3/2}} \end{bmatrix}$$

and the vector of constants,

$$C \begin{bmatrix} D_5 \\ \\ D_6 \end{bmatrix} = \begin{bmatrix} \frac{Y_0}{n}\sum_{i=1}^{n} R_i \frac{x_i y_i}{(x_i^2+y_i^2)^{3/2}} - \frac{X_0}{n}\sum_{i=1}^{n} R_i \frac{y_i^2}{(x_i^2+y_i^2)^{3/2}} + \frac{1}{n}\sum_{i=1} \frac{x_i}{(x_i^2+y_i^2)^{\frac{1}{2}}} + \frac{1}{n}\sum_{i=1}^{n} A_i \\ \\ \frac{X_0}{n}\sum_{i=1}^{n} R_i \frac{x_i y_i}{(x_i^2+y_i^2)^{3/2}} - \frac{Y_0}{n}\sum_{i=1}^{n} R_i \frac{x_i^2}{(x_i^2+y_i^2)^{3/2}} + \frac{1}{n}\sum_{i=1}^{n} \frac{y_i}{(x_i^2+y_i^2)^{\frac{1}{2}}} + \frac{1}{n}\sum_{i=1}^{n} B_i \end{bmatrix}$$

were applied in the following manner;

$$X = \frac{\begin{vmatrix} D_5 & D_3 \\ D_6 & D_4 \end{vmatrix}}{\begin{vmatrix} D_1 & D_3 \\ D_2 & D_4 \end{vmatrix}} = \frac{D_5 \cdot D_4 - D_3 \cdot D_6}{D_1 \cdot D_4 - D_3 \cdot D_2} \tag{10}$$

$$Y = \frac{\begin{vmatrix} D_1 & D_5 \\ D_2 & D_6 \end{vmatrix}}{\begin{vmatrix} D_1 & D_3 \\ D_2 & D_4 \end{vmatrix}} = \frac{D_1 \cdot D_6 - D_5 \cdot D_2}{D_1 \cdot D_4 - D_3 \cdot D_2} \tag{11}$$

Equations 10 and 11 replaced equations 6 and 7 in the iterative process. This method proved highly successful as a rapid convergence process. The results of the analysis of this process are represented by the figures in Appendix A.

With the success of the second method an attempt was made to interpret the distinctive improvement. The coefficient matrix, H, was broken down and analyzed;

$$H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{n} \sum\limits_{i=1}^{n} R_i \frac{y_i^2}{(x_i^2+y_i^2)^{3/2}} & \frac{1}{n} \sum\limits_{i=1}^{n} R_i \frac{x_i y_i}{(x_i^2+y_i^2)^{3/2}} \\ \frac{1}{n} \sum\limits_{i=1}^{n} R_i \frac{x_i y_i}{(x_i^2+y_i^2)^{3/2}} & -\frac{1}{n} \sum\limits_{i=1}^{n} R_i \frac{x_i^2}{(x_i^2+y_i^2)^{3/2}} \end{bmatrix}$$

The elements of the second term were then factored;

$$\frac{1}{n}\begin{bmatrix} -\sum_{i=1}^{n} \frac{R_i}{(x_i^2+y_i^2)^{\frac{1}{2}}}(\sin^2\theta_i) & \sum_{i=1}^{n} \frac{R_i}{(x_i^2+y_i^2)^{\frac{1}{2}}}(\sin\theta_i\cos\theta_i) \\ \sum_{i=1}^{n} \frac{R_i}{(x_i^2+y_i^2)^{\frac{1}{2}}}(\sin\theta_i\cos\theta_i) & -\sum_{i=1}^{n} \frac{R_i}{(x_i^2+y_i^2)^{\frac{1}{2}}}(\cos^2\theta_i) \end{bmatrix}$$

The matrix below extracted from the second term of the coefficient matrix has a determinant whose value is equal to zero for any values of θ_i;

$$\begin{bmatrix} -\sin^2\theta_i & \sin\theta_i\cos\theta_i \\ \sin\theta_i\cos_i & -\cos^2\theta_i \end{bmatrix}$$

If, in fact, there were no errors between the measured ranges and the actual ranges then $R_i = (x_i^2 + y_i^2)^{\frac{1}{2}}$ and the factored term, $\frac{R_i}{(x_i^2 + y_i^2)^{\frac{1}{2}}}$, would equal one. This implies that if there were no error between the measured and actual ranges, the second term in the matrix H would go to zero leaving;

$$H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

It appears that the existing error between the actual and measured ranges causes variations in the value of the second term in the coefficient matrix H and therefore provides the driving force of the convergence process.

VI. GROUND SPEED DETERMINATION

From the problem description in Section IV, the mathematical model below for ground speed determination was developed. Figure 5 provided the geometry for the problem. The particular geometry in the figure indicates three range measurements taken at equal intervals. Ground speed, using from three to eight ranges measurements was developed in this study. The distances C_i calculated below are inputs to the ground speed determination equations 18 and 19 at the end of this section where $i = 3,4,\ldots,8$ represents the number of measured ranges utilized in the calculation.

For the three range geometry the known values are R_1, R_2 and R_3. It is also assumed that the time between range measurements is the same in each case. The unknown values are X, Y and C. The three equations from which C was determined are applications of the Pythagorean theorem.

$$(X + 2C)^2 + Y^2 = R_1^2 \rightarrow X^2 + 4CX + 4C^2 + Y^2 = R_1^2 \quad \text{(a)}$$
$$(X^2 + C)^2 + Y^2 = R_2^2 \rightarrow X^2 + 2CX + C^2 + Y^2 = R_2^2 \quad \text{(b)}$$
$$X^2 + Y^2 + R_3^2 \rightarrow X^2 + Y^2 = R_3^2 \quad \text{(c)}$$

Equations (a) plus (c) minus 2(b);

$$\begin{array}{rcl}
X^2 + 4CX + 4C^2 + Y^2 & = & R_1^2 \\
X^2 \qquad\qquad\quad + Y^2 & = & R_3^2 \\
-2X^2 - 4CX - 2C^2 - 2Y^2 & = & -2R_3^2 \\
\hline
2C^2 & = & R_1^2 - 2R_2^2 + R_3^3
\end{array}$$

$$C_3 = \sqrt{\frac{R_1^2 - 2R_2^2 + R_3^2}{2}} \qquad (12)$$

The development for four ranges;

$$(X + 3C)^2 + Y^2 = R_1^2 \rightarrow X^2 + 6CX + 9C^2 + Y^2 = R_1^2 \quad \text{(a)}$$

$$(X + 2C)^2 + Y^2 = R_2^2 \rightarrow X^2 + 4CX + 4C^2 + Y^2 = R_2^2 \quad \text{(b)}$$

$$(X + C)^2 + Y^2 = R_3^2 \rightarrow X^2 + 2CX + C^2 + Y^2 = R_3^2 \quad \text{(c)}$$

$$X^2 + Y^2 = R_4^2 \rightarrow X^2 + Y^2 = R_4^2 \quad \text{(d)}$$

Equations (a) minus (b) minus (c) plus (d);

$$\begin{array}{rcl} X^2 + 6CX + 9C^2 + Y^2 & = & R_1^2 \\ -X^2 - 4CX - 4C^2 - Y^2 & = & R_2^2 \\ -X^2 - 2CX - C^2 - Y^2 & = & R_3^2 \\ X^2 + Y^2 & = & R_4^2 \\ \hline 4C^2 & = & R_1^2 - R_2^2 - R_3^2 + R_4^2 \end{array}$$

$$C_4 = \sqrt{\frac{R_1^2 - R_2^2 - R_3^2 + R_4^2}{4}} \quad (13)$$

The development for five through eight ranges was omitted but was similar to the two cases above;

$$C_5 = \sqrt{\frac{2R_1^2 - 3R_2^2 + R_3^2 - R_4^2 + R_5^2}{8}} \quad (14)$$

$$C_6 = \sqrt{\frac{2R_1^2 - 2R_2^2 - R_3^2 + R_4^2 - R_5^2 + R_6^2}{12}} \quad (15)$$

$$C_7 = \sqrt{\frac{R_1^2 - R_2^2 + R_3^2 - 2R_4^2 + R_5^2 - R_6^2 + R_7^2}{12}} \quad (16)$$

$$C_8 = \sqrt{\frac{R_1^2 - R_2^2 + R_3^2 - R_4^2 - R_5^2 + R_6^2 - R_7^2 + R_8^2}{16}} \quad (17)$$

The value C_i is a distance in units of nautical miles or yards in each of the cases above.

The conversion to ground speed was easily accomplished; (1) for C_i in nautical miles and (2) for C_i in yards;

$$(1)\quad GS = \frac{C_i\,(nm)}{TIME\,(sec)} \cdot \frac{3600\,(sec)}{1\,(hr)} = \frac{3600 \cdot C_i}{TIME}\ \text{knots} \tag{18}$$

$$(2)\quad GS = \frac{C_i\,(yds)}{TIME\,(sec)} \cdot \frac{3600\,(sec)}{1\,(hr)} \cdot \frac{1\,(nm)}{2000\,(yds)} = \frac{18 \cdot C_i}{TIME}\ \text{knots} \tag{19}$$

where TIME is the time interval in seconds between each range measurement.

VII. TRUE COURSE DETERMINATION

The true course determination used the ground speed as calculated in the previous section. Figures 6 and 7 of Section V provided the geometry for the problem and Figure 7-b is annotated for the following mathematical development. The coordinates of X and Y were first determined. This was done by the simultaneous solution of a pair of quadratic equations.

$$(X-a)^2 + (Y-b)^2 = G_N^2$$

$$(X-c)^2 + (Y-d)^2 = G_E^2$$

Expanding these equations;

$$X^2 - 2aX + a^2 + Y^2 - 2bY + b^2 = G_N^2 \qquad \text{(a)}$$

$$X^2 - 2cX + C^2 + Y^2 - 2dY + d^2 = G_E^2 \qquad \text{(b)}$$

Subtract (b) from (a);

$$2(c-a)X + a^2 - c^2 + 2(d-b)Y + b^2 - d^2 = G_N^2 - G_E^2$$

Solve for Y in terms of X;

$$2(d-b)Y = 2(a-c)X + G_N^2 - G_E^2 + c^2 + d^2 - a^2 - b^2$$

$$Y = \frac{(a-c)}{(d-b)} \cdot X + \frac{G_N^2 - G_E^2 + c^2 + d^2 - a^2 - b^2}{2(d-b)}$$

Let

$$R = \frac{a-c}{d-b} \quad \text{and} \quad S = \frac{G_N^2 - G_E^2 + c^2 + d^2 - a^2 - b^2}{2(d-b)}$$

Then

$$Y = R \cdot X + S \qquad (20)$$

Substitute the value for Y in equation (b) above;

$$X^2 - 2cX + c^2 + (R \cdot X+S)^2 - 2d(R \cdot X+S) + d^2 = G_N^2$$

$$X^2 - 2cX + c^2 + R^2X^2 + 2RSX + S^2 - 2dRX - 2dS + d^2 - G_E^2 = 0$$

Collecting terms;

$$(1+R^2)X^2 + (2RS-2c-2dR)X + (c^2+d^2+S^2-2dS-G_E^2) = 0$$

This is a quadratic in X;

$$X = \frac{-B + \quad B^2-4AC}{2A} \tag{21}$$

where

$$A = (1+R^2)$$

$$B = (2RS-2c-2dR)$$

$$C = (c^2+d^2+S^2-2dS-G_E^2)$$

Equation (18) is a quadratic form providing two possible solutions. The values of the coordinates X and Y determined by both solutions of X provide points which are mirror images on each side of a line joining the points (a,b) and (c,d) in Figure 7-b. The choice of coordinates (X,Y) depends on the initial heading of the aircraft and whether a right or left turn is executed on the second leg.

From the coordinates (X,Y) the drift vector was then determined. The coordinates (e,f) are known.

$$W = \sqrt{(X-e)^2 + (Y-f)^2} \tag{22}$$

The drift vector was then applied to the terminal heading to determine the true course of the aircraft for accurate track determination.

VIII. CONCLUSIONS

The study showed that the method of determining sonobuoy location relative to the aircraft with consecutive, short-interval range measurement techniques is feasible. Moreover the application of the principle of the first derivative of the range with respect to time of a buoy near the aircraft heading as a means for determining ground speed is accurate and relatively insensitive to range errors, number of ranges used and the relative angle of the buoy.

An increase in the number of consecutive ranges measured in excess of three, reduces the error in buoy location but the reduction is slight. The aircraft should not sacrifice flexibility to acquire additional range measurements unless tactics permit.

In the sensitivity analysis of the sonobuoy location technique, the convergence process appeared to break down for acute relative buoy angles where the estimate of the relative angle lay between the actual relative angle and the aircraft heading. Before implementation of this system further research should be conducted in this area to possibly provide a 360° zone of reliable information on buoy location. The model, at present, can provide 360° of reliable information. For acute relative buoy angles, with respect to the aircraft heading, however, the estimated angle must be greater than the actual angle.

It is felt that the utilization of buoy ranging information for the location of sonobuoys, apart from

triangulation procedures, has a broad field of application, yet uncovered. A preliminary study was conducted to determine relative buoy positions independent of aircraft navigation. Ranges to all buoys in the pattern were measured from a single aircraft position. For a pattern consisting of N buoys there were $\frac{N!}{(2!)(N-2)!}$ possible two-buoy combinations. The actual buoy positions were not known thus the assumed positions were used. The ranges were plotted in reverse from the assumed positions of the buoys generating $\frac{N!}{(2!)(N-2)!}$ aircraft fix positions determined by the ranges from each two-buoy combination. From these resulting fixes an average position was determined by least squares method. An iterative process was then developed whereby the assumed buoy positions were adjusted such that the reverse plotting of the ranges would generate $\frac{N!}{(2!)(N-2)!}$ aircraft fixes that were coincident. This did not produce a unique set of final assumed positions for the buoys but did establish a locus for each buoy in the pattern relative to the aircraft position. It is felt that further study in this area would provide, possibly, a more versatile use of sonobuoy ranging.

APPENDIX A, SONOBUOY LOCATION ERROR ANALYSIS

The first study conducted with respect to sonobuoy location was to determine the response of the speed of convergence of the iterative process as affected by the variance of;

(1) The relative angle of the buoy with respect to the aircraft heading, θ.

(2) Varying estimates, θ_o, of the relative angle.

(3) Range from the aircraft to the buoy.

Figures 8 through 11 show the effect on the number of iterations required to converge by variance in the estimate of the relative angle of the buoy. The actual relative angle ranged from 10^o to 80^o in ten degree increments. The estimate of the relative angle, in all but one case, ranged from -15^o to $+15^o$ in error of the actual relative angle in increments of one degree. For the case where the relative buoy angle equals 10^o the angle estimate, θ_o, started at 0^o which was -10^o in error of the actual angle. The convergence is quite rapid for all estimates, θ_o, particularly when $\theta_o \approx \theta$. The convergence process breaks down when the buoy is close to the heading of the aircraft and an estimate θ_o, between the actual relative angle and the aircraft heading is chosen. This can be seen in Figure 8. It should be noted that for any relative angle, θ, however small, if an estimate, θ_o, is chosen which is less acute than θ, the process provides rapid convergence. Data for the effect on the speed of convergence for variance in buoy range is

not included. The effect was relatively insignificant. Buoy ranges at 30 nm. and 5 nm. were tested with at most one iteration difference (in favor of the 30 nm. range) in the convergence. The data in Figures 8 through 11 represent a buoy range of approximately 15 nm. and a range error of zero. The convergence process stopped when the buoy location error was within .5% of the range to the buoy.

The second study incorporated a range error to determine its effect on final buoy location at the termination of the convergence process. Relative buoy angles of 20°, 45° and 80° were investigated. Figures 12, 13 and 14 show the total buoy location error in yards versus the application of the range error in the following manner to the three ranges;

R	R	R	
0	0	0	no error applied
+	+	+	all errors added
-	-	-	all errors subtracted
+	-	+	other combinations
-	+	-	other combinations

There appears to be an approximate one-to-one relationship of the average buoy location error to the range error applied which is appealing with respect to the small angles utilized between range measurements.

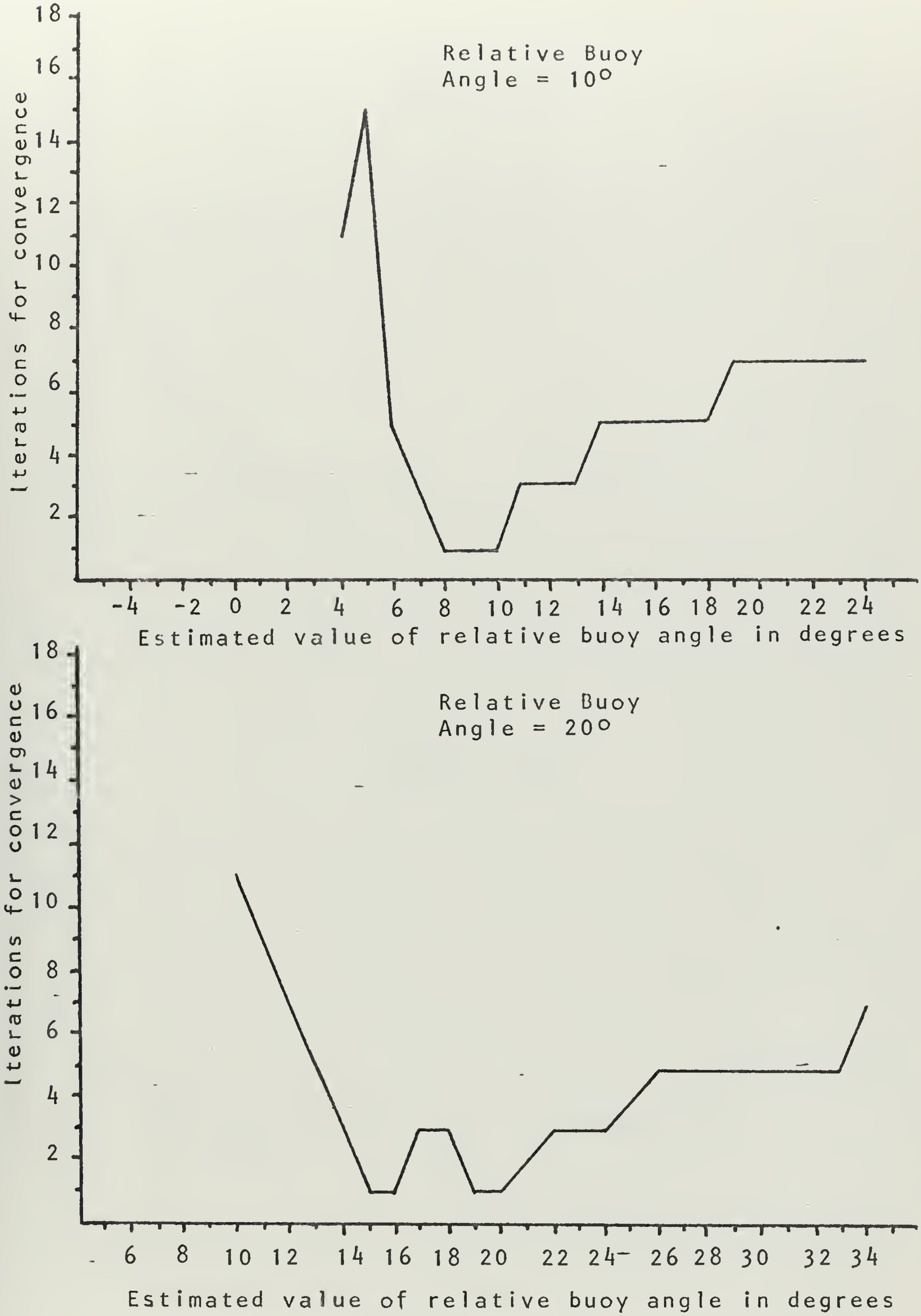


FIGURE 8

NUMBER OF ITERATIONS REQUIRED FOR CONVERGENCE VERSUS ESTIMATE OF RELATIVE BUOY ANGLE THETA

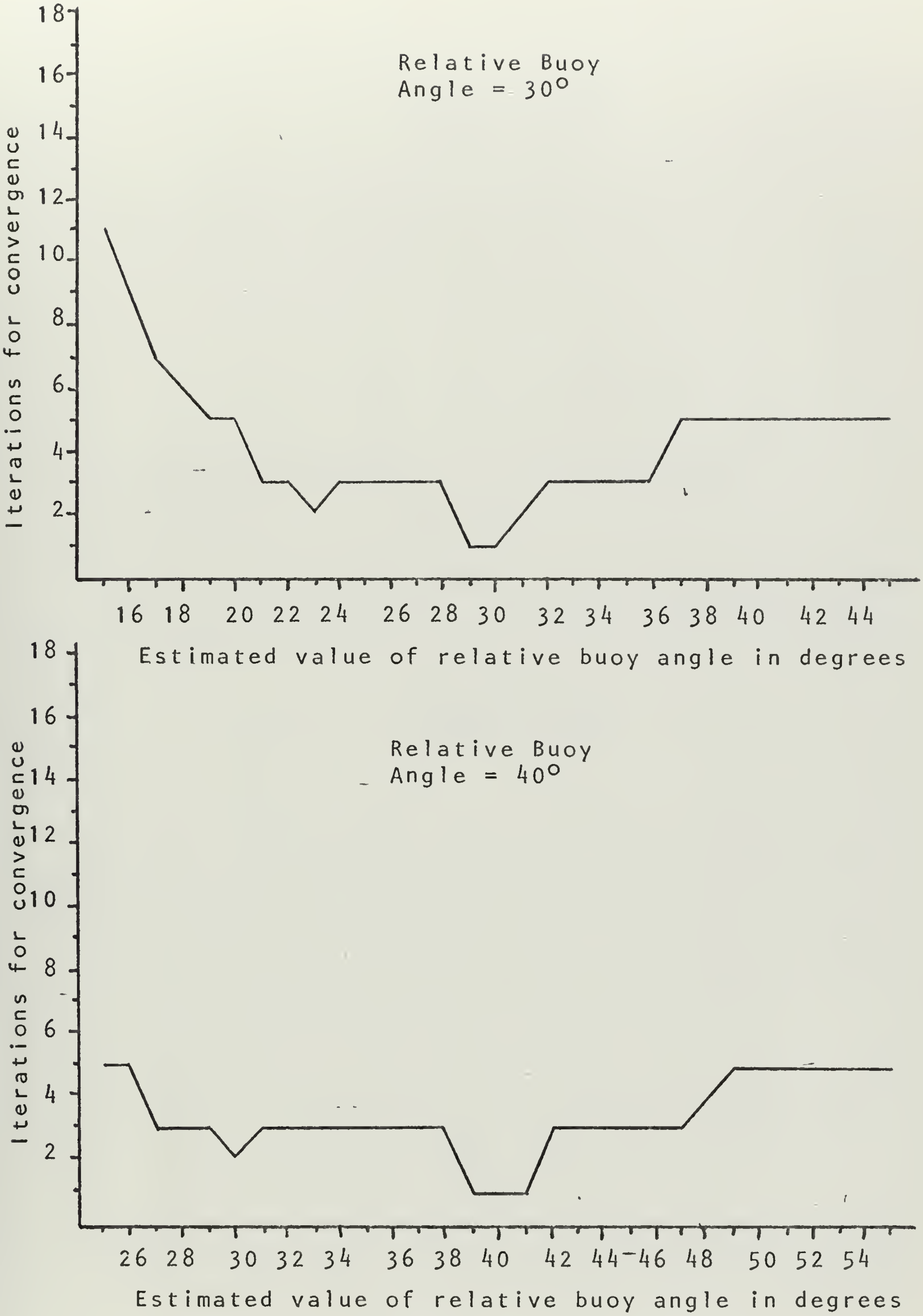


FIGURE 9

NUMBER OF ITERATIONS REQUIRED FOR CONVERGENCE VERSUS ESTIMATE OF RELATIVE BUOY ANGLE THETA

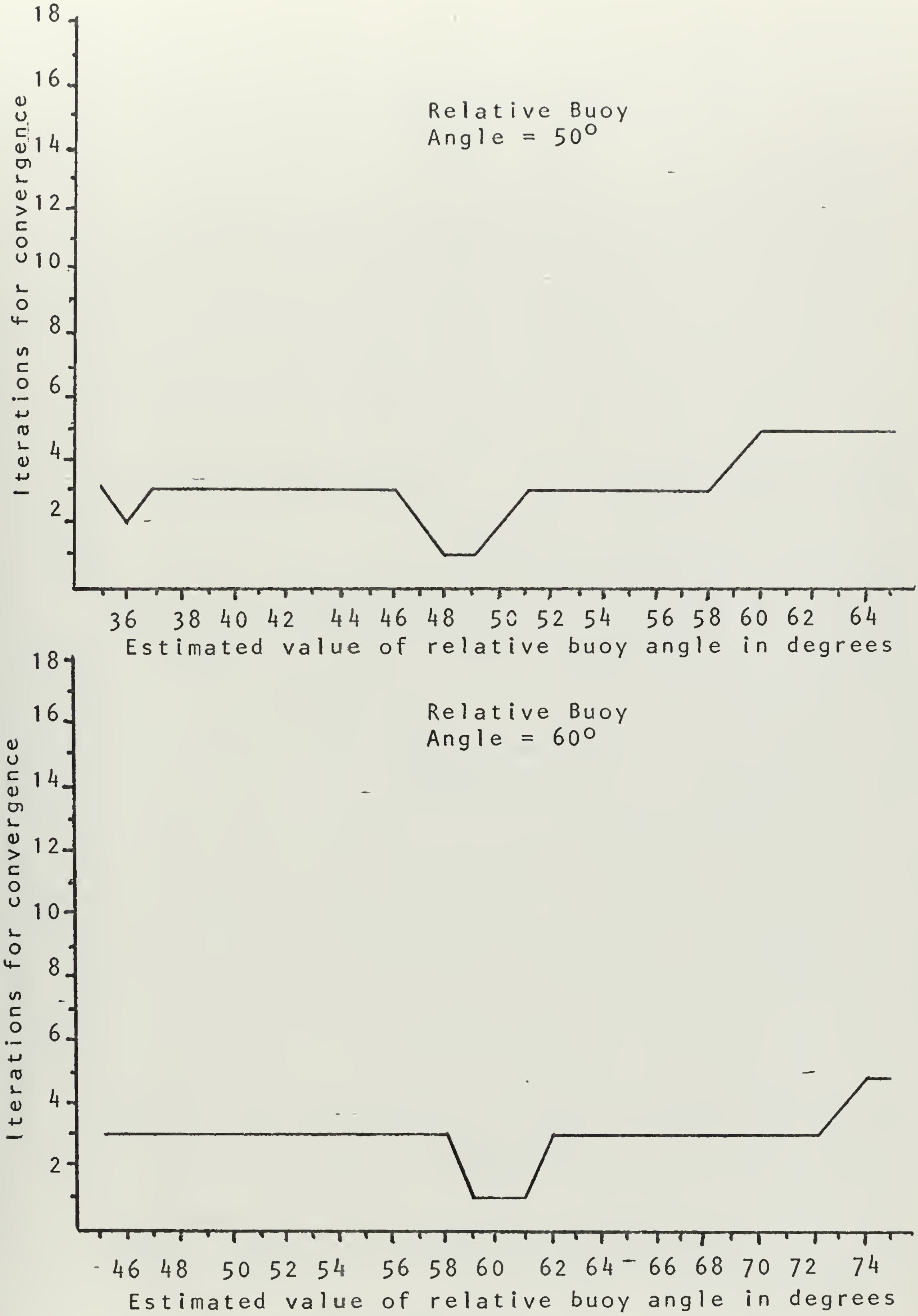
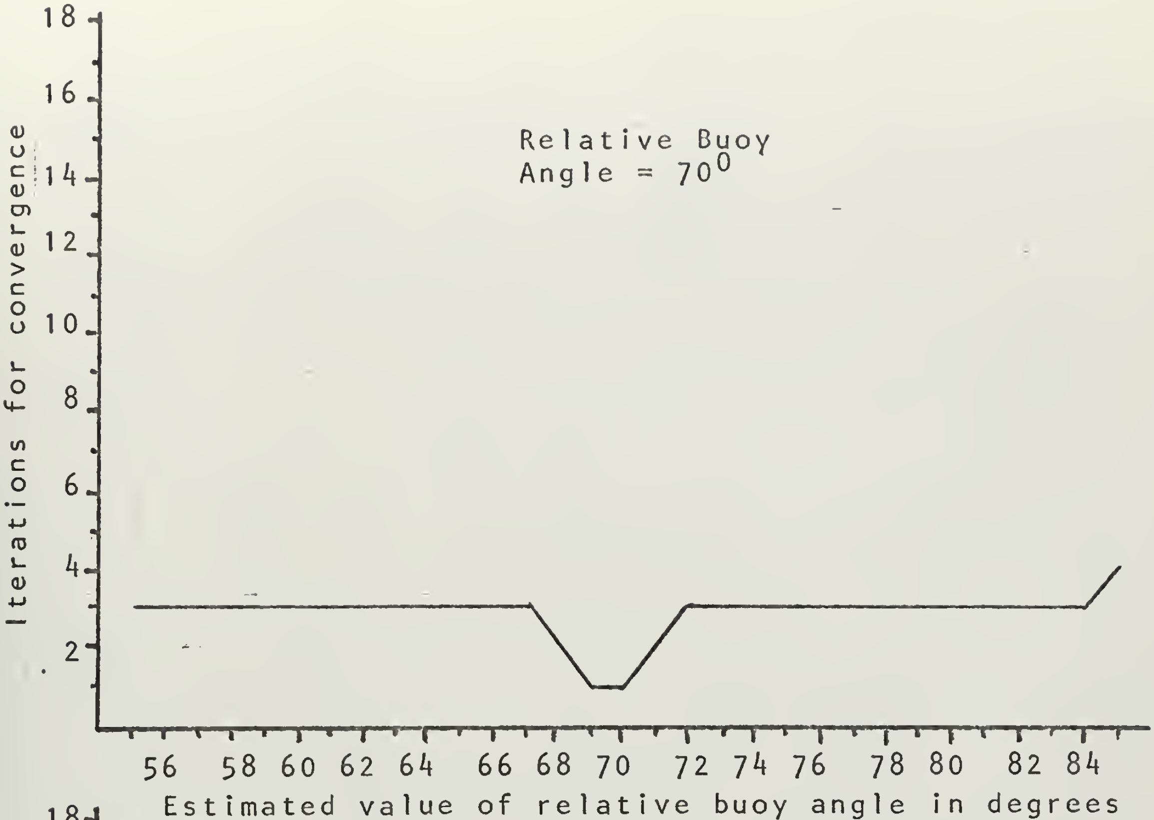


FIGURE 10

NUMBER OF ITERATIONS REQUIRED FOR CONVERGENCE VERSUS ESTIMATE OF RELATIVE BUOY ANGLE THETA



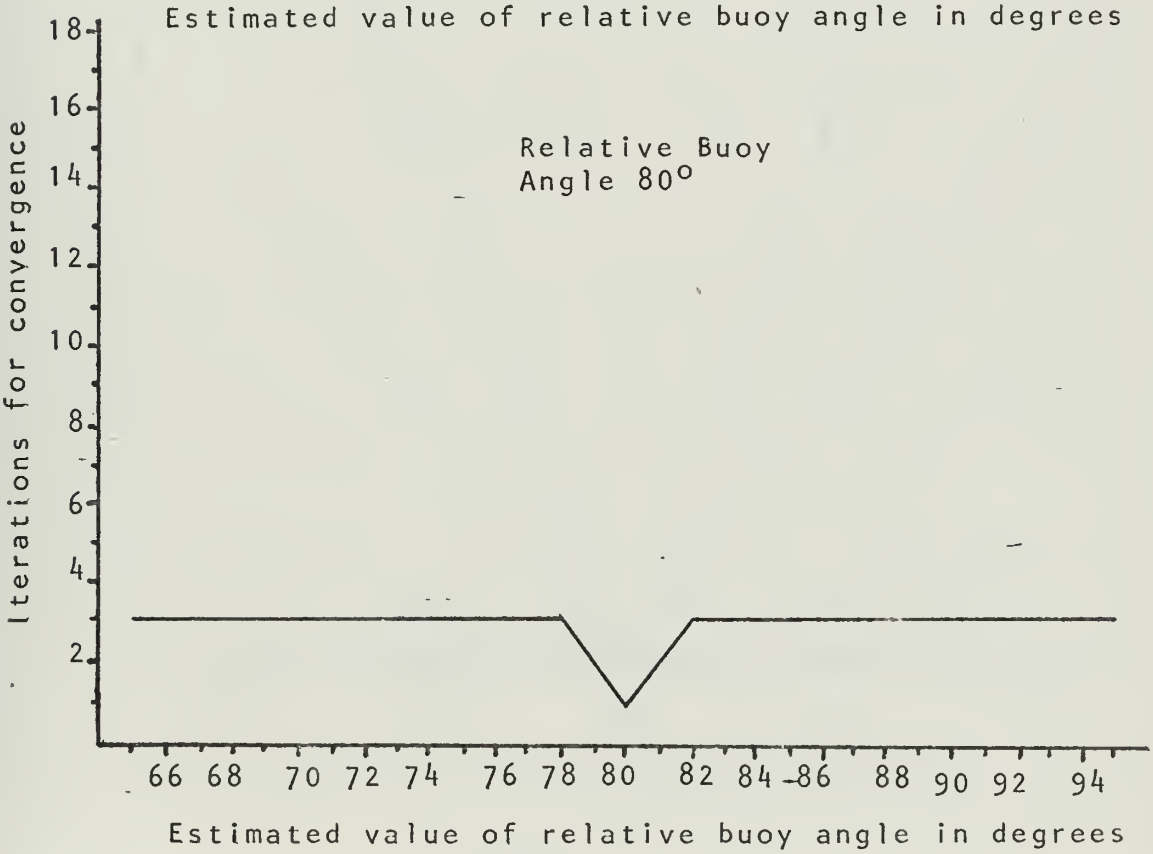


FIGURE 11

NUMBER OF ITERATIONS REQUIRED FOR CONVERGENCE VERSUS ESTIMATE OF RELATIVE BUOY ANGLE THETA

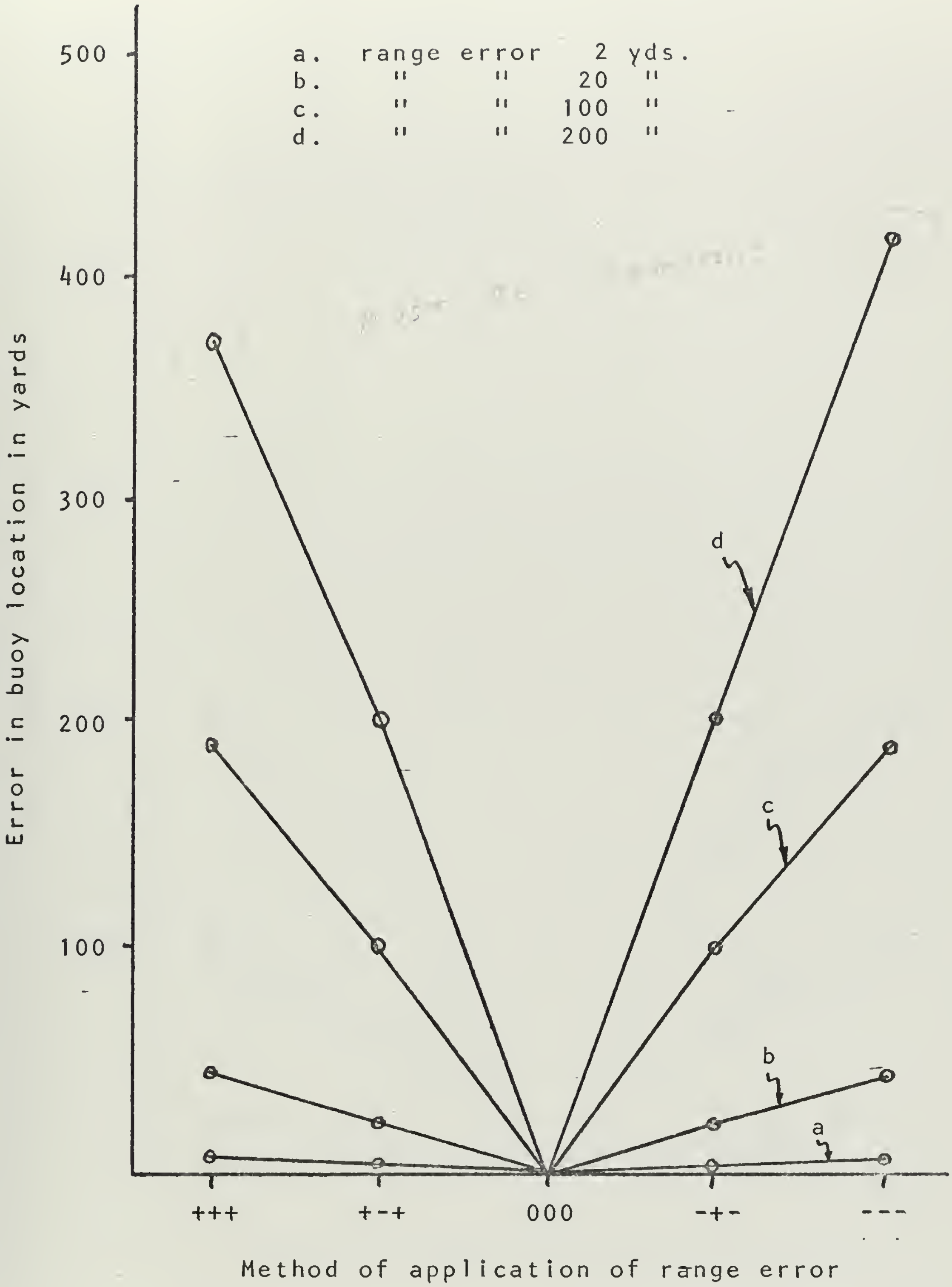


FIGURE 12

ERROR IN BUOY LOCATION IN YARDS VERSUS METHOD OF APPLICATION
RELATIVE BUOY ANGLE 20°

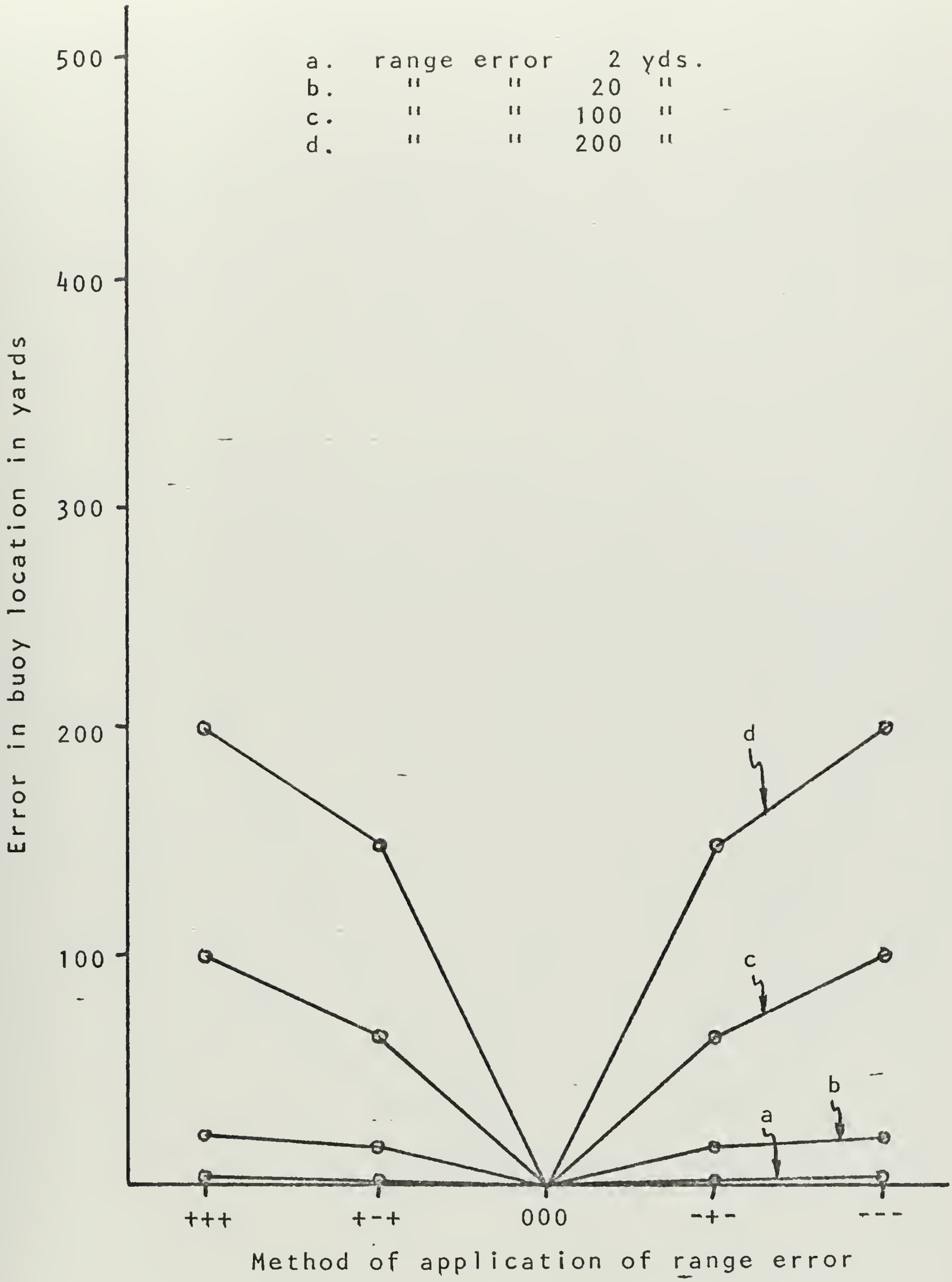


FIGURE 13

ERROR IN BUOY LOCATION IN YARDS VERSUS METHOD OF APPLICATION
RELATIVE BUOY ANGLE 45°

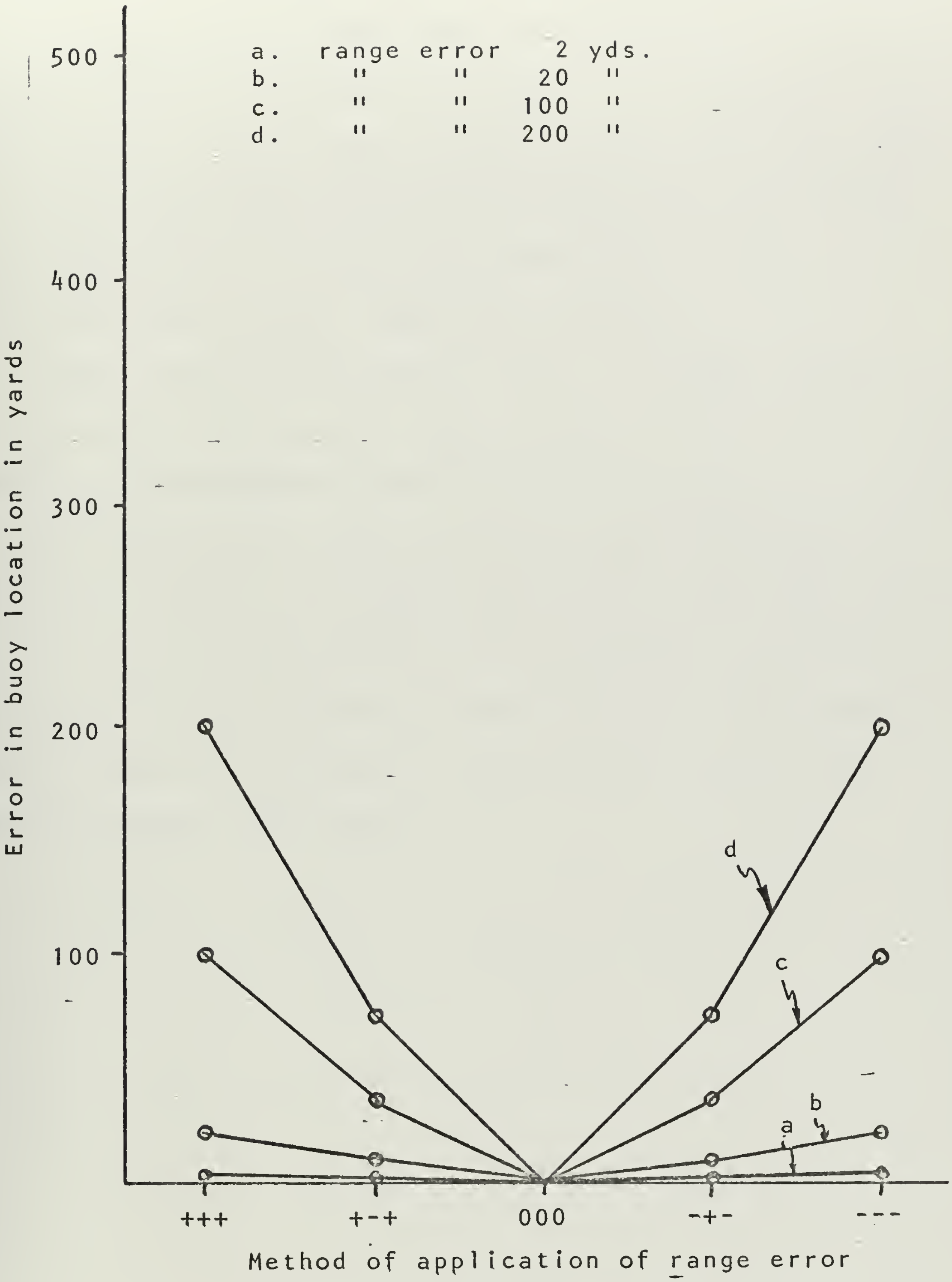


FIGURE 14

ERROR IN BUOY LOCATION IN YARDS VERSUS METHOD OF APPLICATION
RELATIVE BUOY ANGLE 80°

APPENDIX B, GROUND SPEED DETERMINATION ERROR ANALYSIS

The results of the study conducted on the ground speed determination are pictured in Figures 15 through 20. They represent the ground speed error in knots versus the range error applied, under the varying conditions listed below;

(1) Number of ranges measured (3,4,...,8).

(2) Time between range measurements.

(3) Relative bearing of buoy to aircraft heading.

In no case does the ground speed exceed 1.75 knots. The case of-the relative buoy angle of zero was not tested. It was felt that this was a trivial case. The change in horizontal range with respect to time of a buoy dead ahead was the ground speed exactly. It was desired to examine the accuracy of the method of application of first derivative of range with respect to time as the relative buoy angle increased from zero.

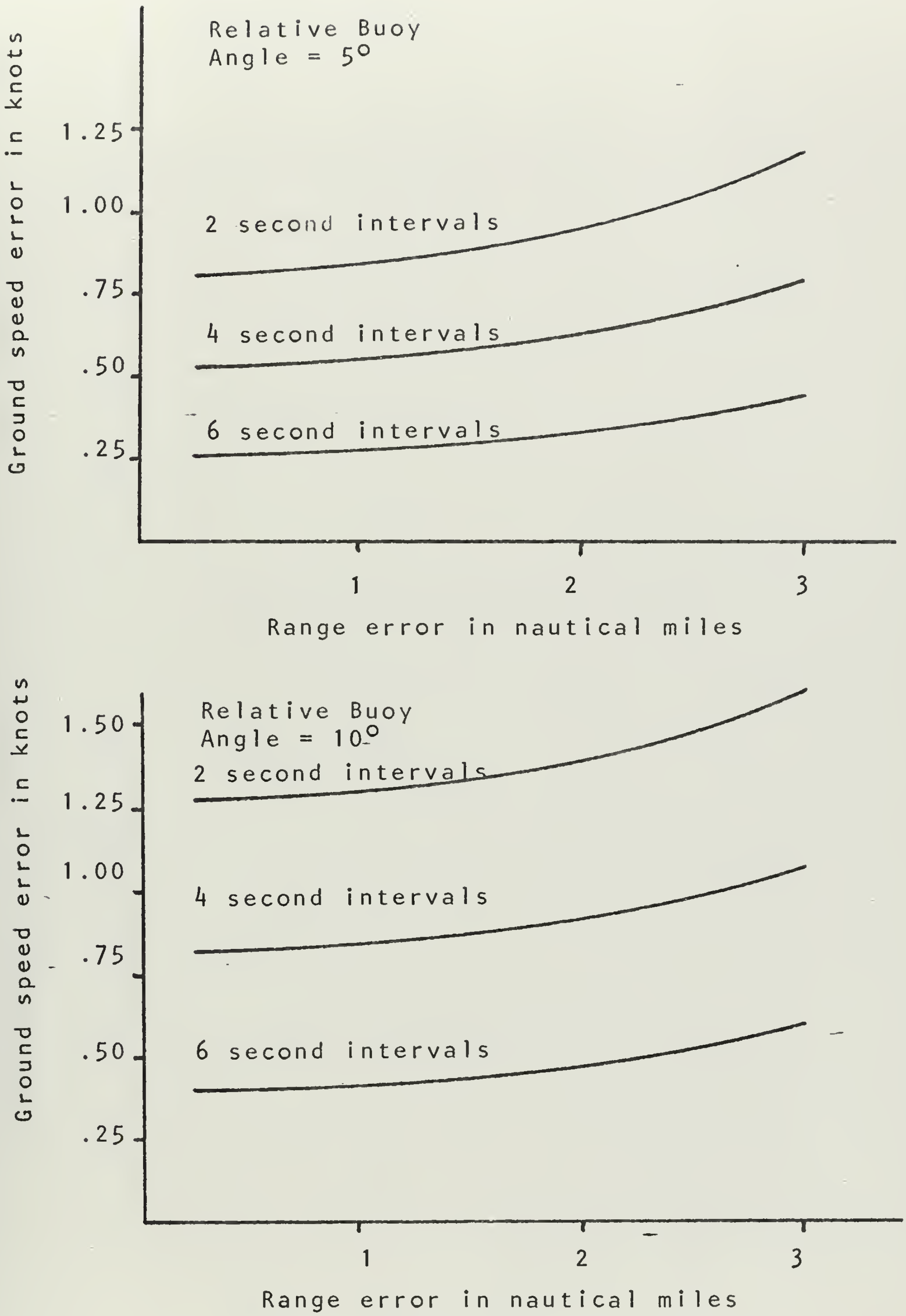


FIGURE 15

GROUND SPEED ERROR IN KNOTS VERSUS ERROR IN RANGE DUE TO DRIFT IN BUOY FREQUENCY WITH RESPECT TO TIME - 3 RANGES

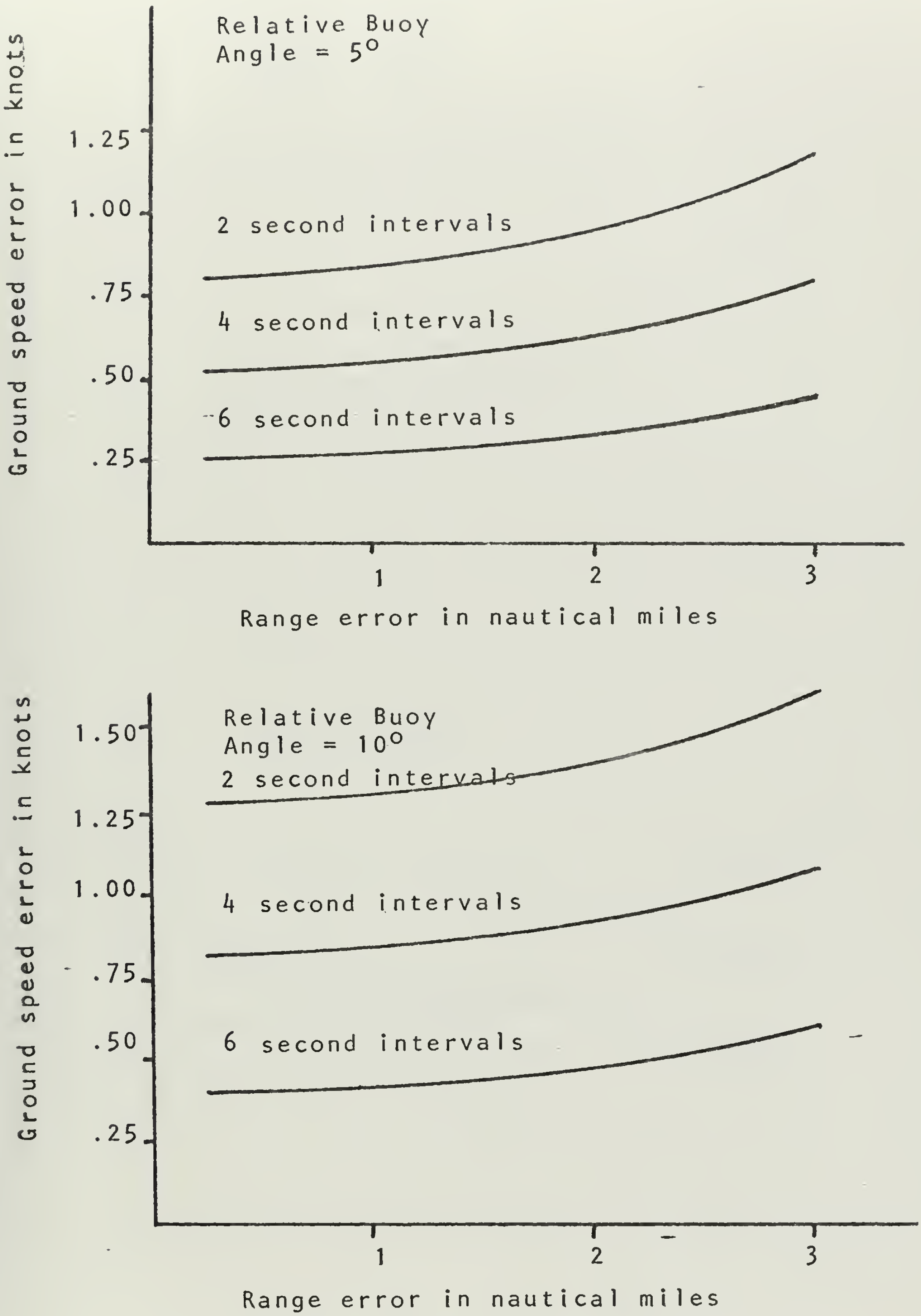


FIGURE 16

GROUND SPEED ERROR IN KNOTS VERSUS ERROR IN RANGE DUE TO DRIFT IN BUOY FREQUENCY WITH RESPECT TO TIME - 4 RANGES

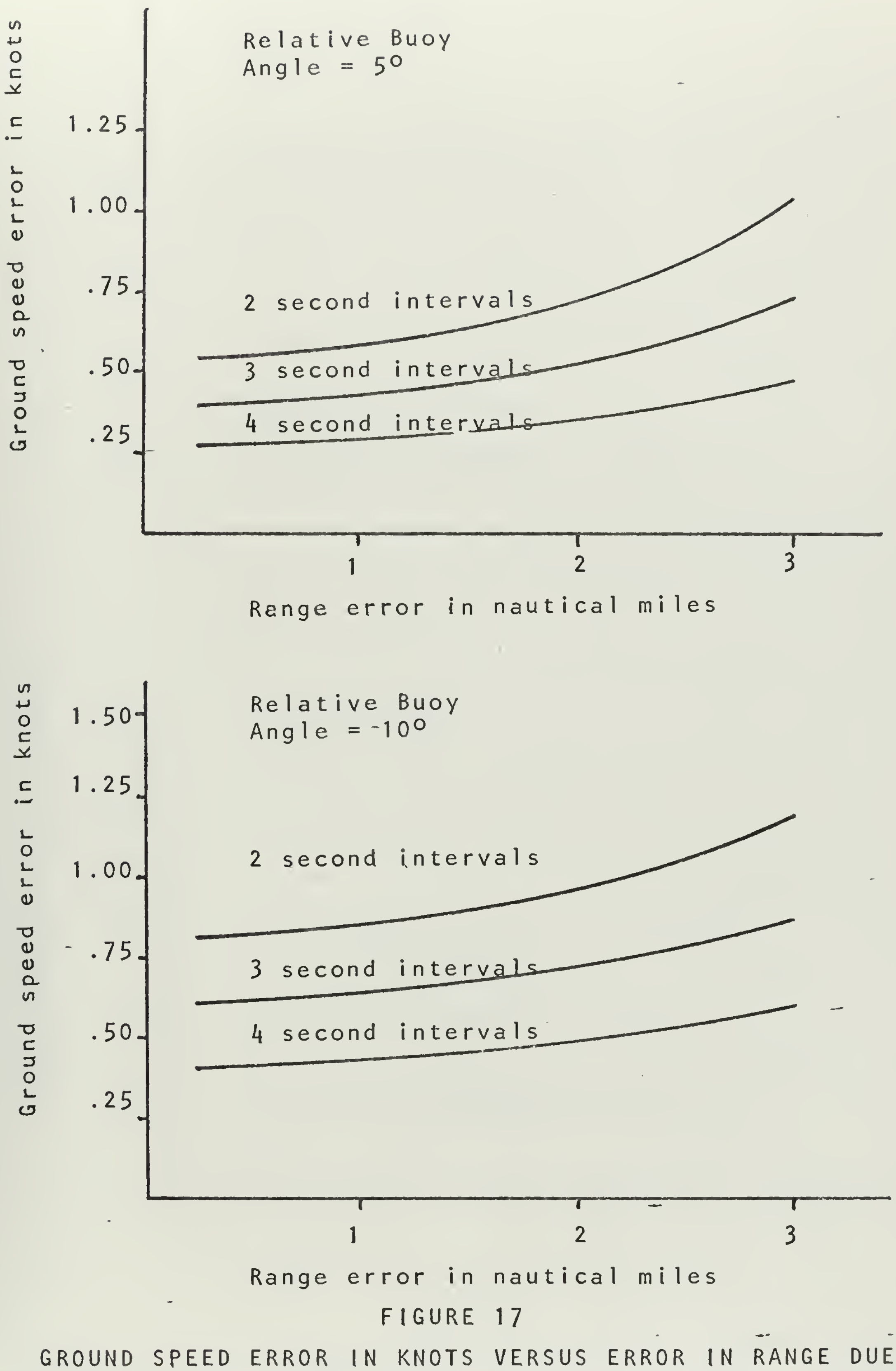


FIGURE 17

GROUND SPEED ERROR IN KNOTS VERSUS ERROR IN RANGE DUE TO DRIFT IN BUOY FREQUENCY WITH RESPECT TO TIME - 5 RANGES

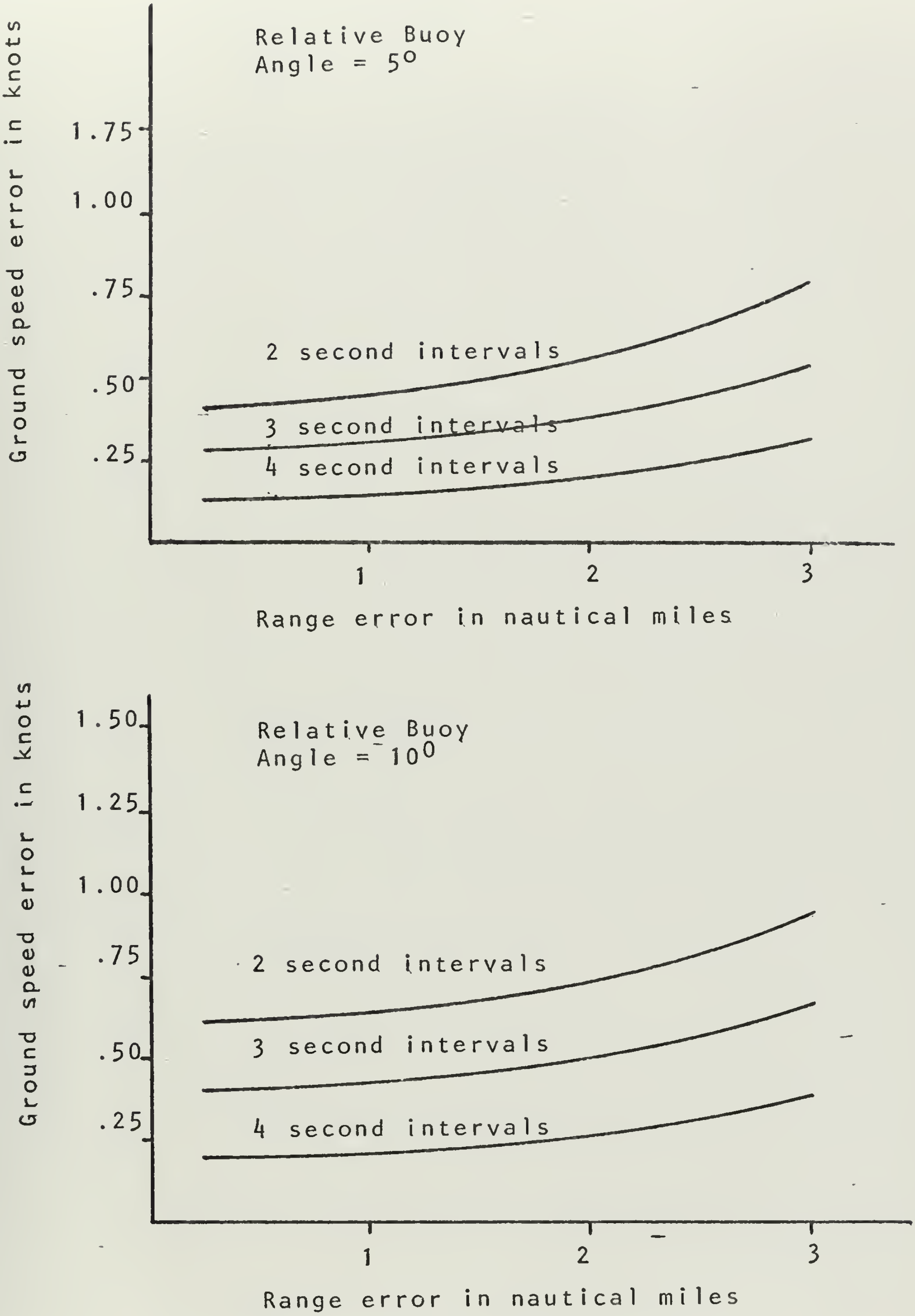


FIGURE 18

GROUND SPEED ERROR IN KNOTS VERSUS ERROR IN RANGE DUE TO DRIFT IN BUOY FREQUENCY WITH RESPECT TO TIME - 6 RANGES

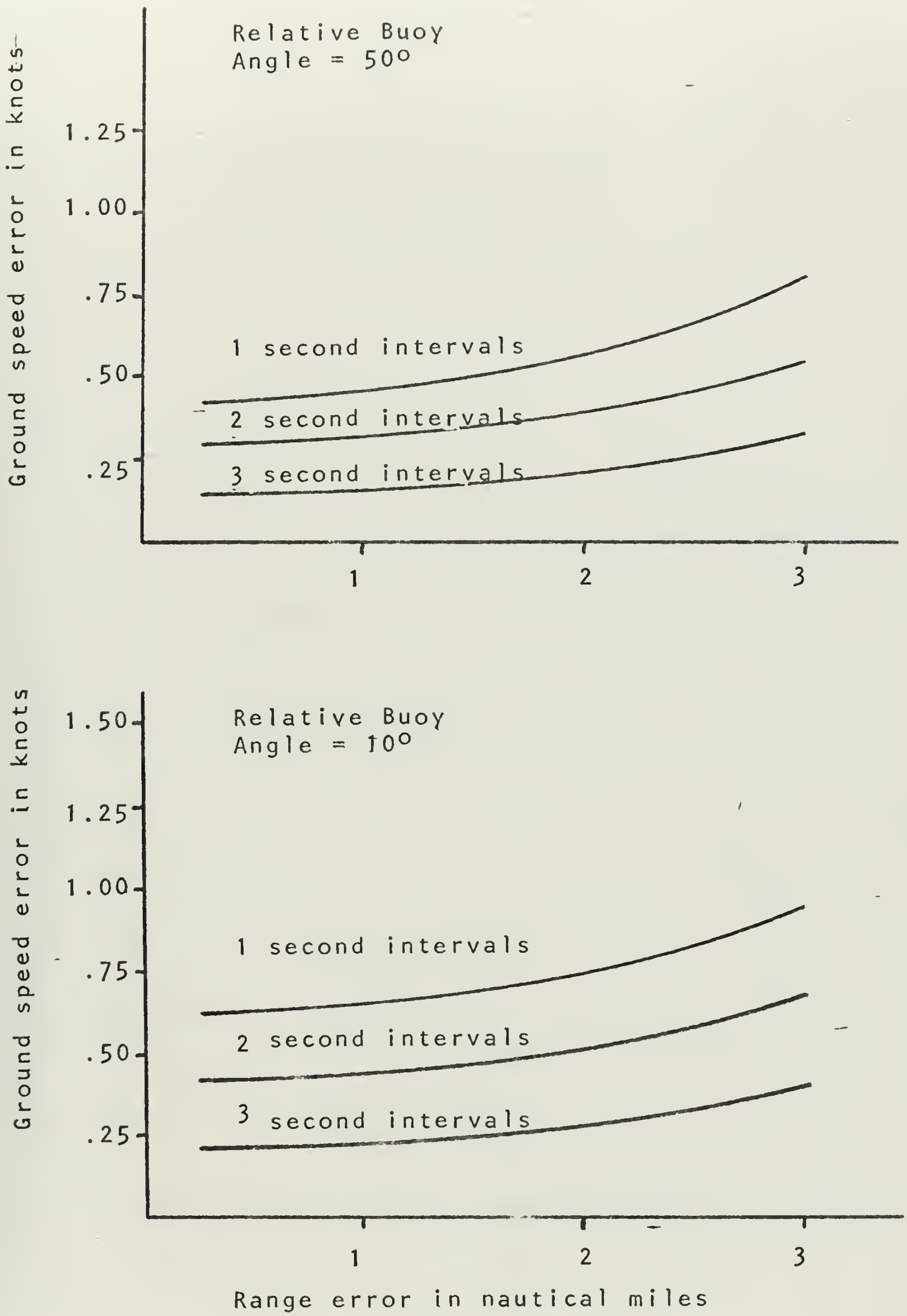


FIGURE 19

GROUND SPEED ERROR IN KNOTS VERSUS ERROR IN RANGE DUE TO DRIFT IN BUOY FREQUENCY WITH RESPECT TO TIME - 7 RANGES

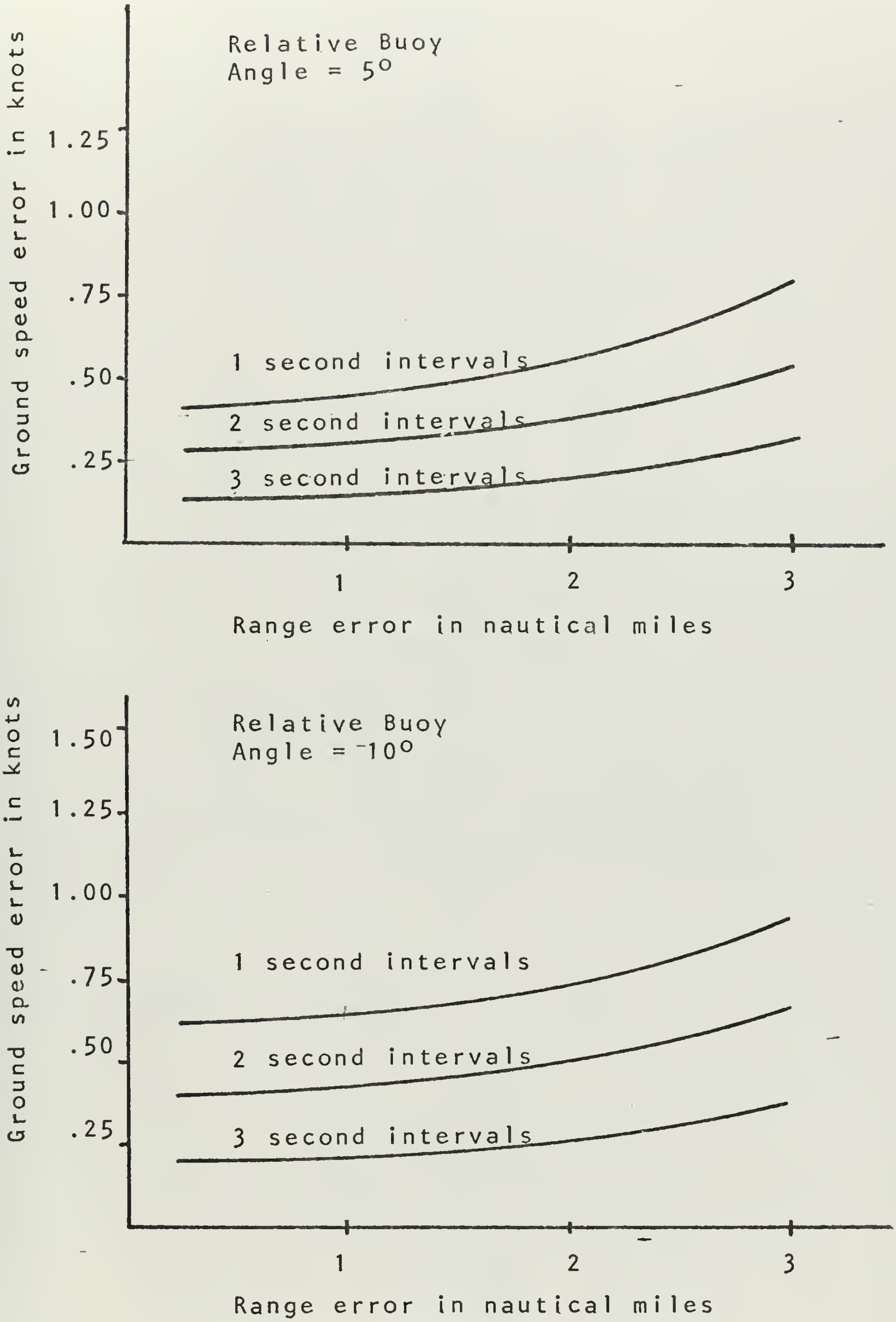
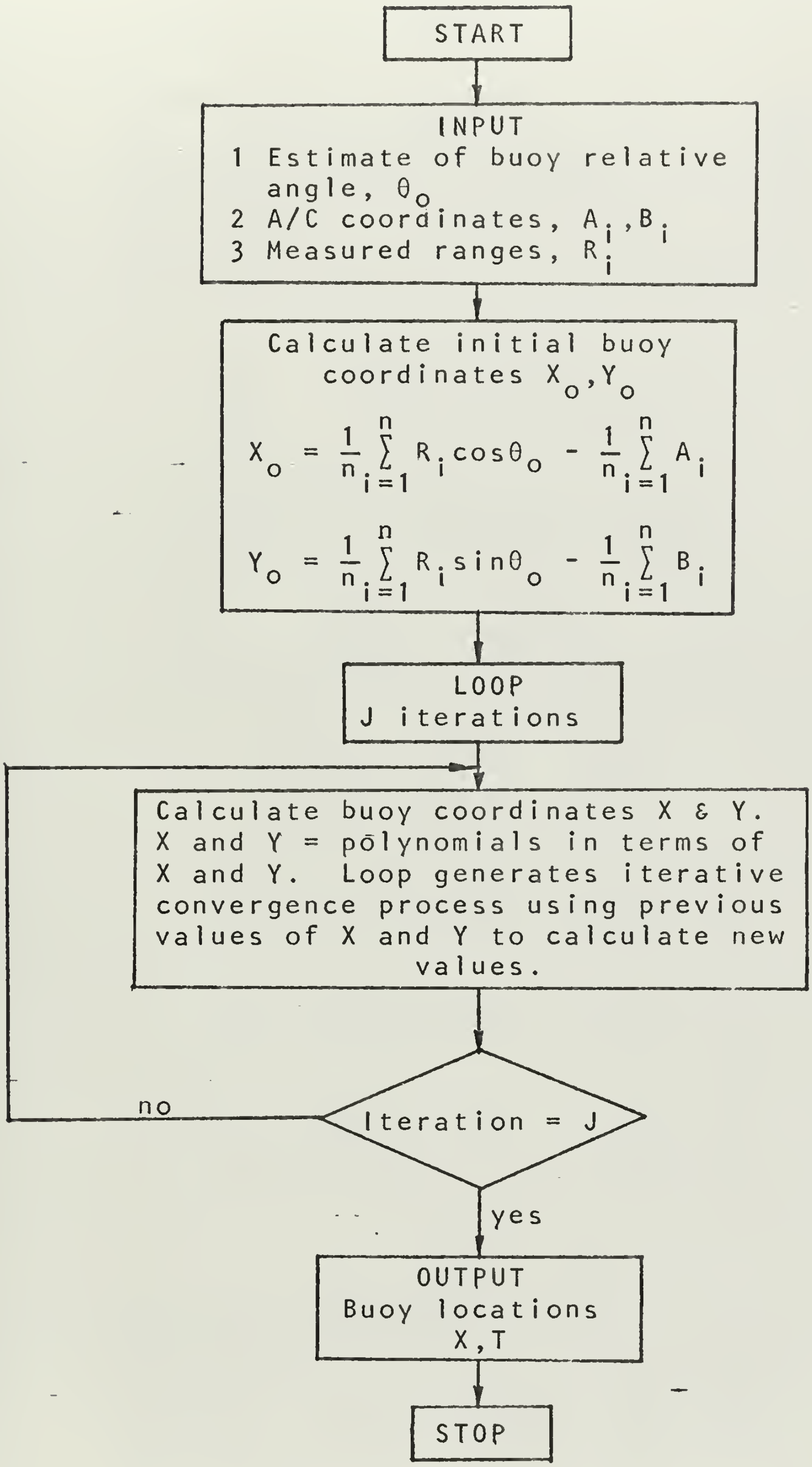


FIGURE 20

GROUND SPEED ERROR IN KNOTS VERSUS ERROR IN RANGE DUE TO DRIFT IN BUOY FREQUENCY WITH RESPECT TO TIME - 8 RANGES

SONOBUOY LOCATION PROGRAM



SONOBUOY LOCATION SIMULATION PROGRAM

START

INPUT
1 A/C coordinates, $(A_i B_i)$
2 Actual buoy coordinates, (C, D)
3 Estimate of relative buoy angle, θ_o

Calculate actual ranges to buoy

$$R_i = \sqrt{(C-A_i)^2 + (D-B_i)^2}$$

Establish acceptable error test value as .5% of range

$$TESTX = .005(R_i)$$

$$TESTY = .005(R_i)$$

$$\sum_{i=1}^{n} A_i, \quad \sum_{i=1}^{n} B_i, \quad \sum_{i=1}^{n} R_i$$

Calculate initial values of buoy coordinates using θ_o

$$X_o = \frac{1}{n}\sum_{i=1}^{n} R_i \cos\theta_o - \frac{1}{n}\sum_{i=1}^{n} A_i$$

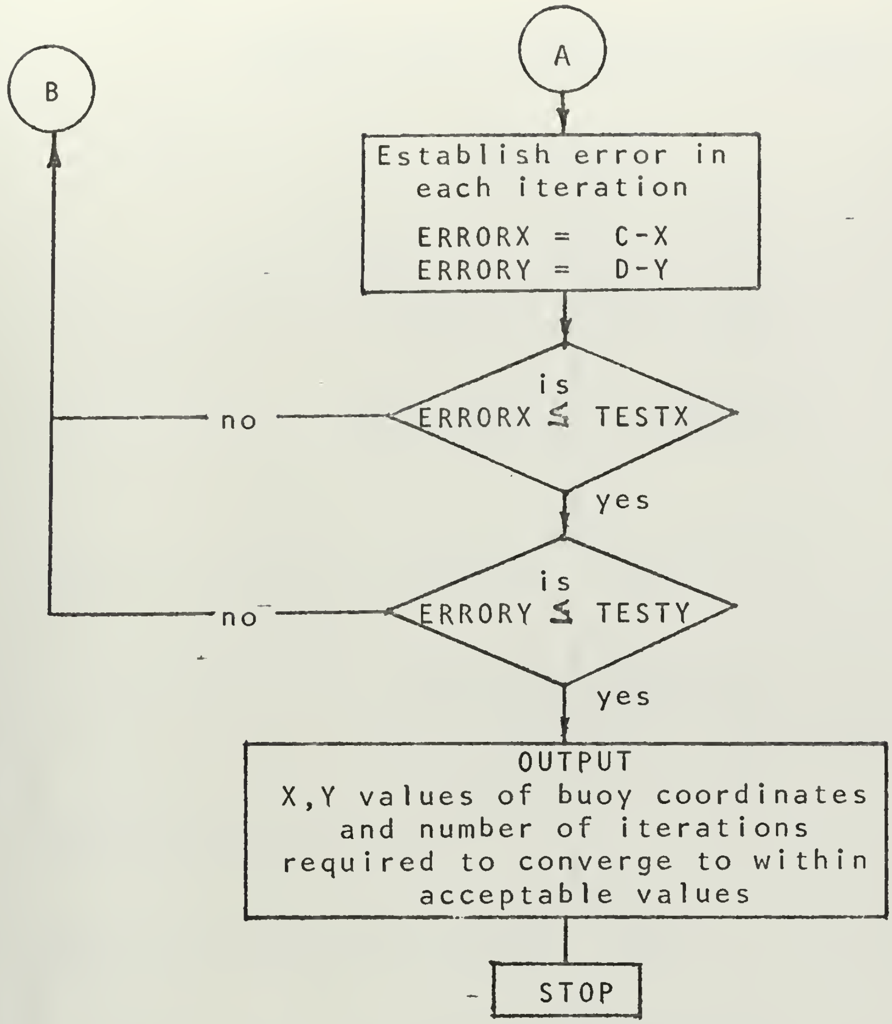
$$Y_o = \frac{1}{n}\sum_{i=1}^{n} R_i \sin\theta_o - \frac{1}{n}\sum_{i=1}^{n} B_i$$

LOOP
J iterations

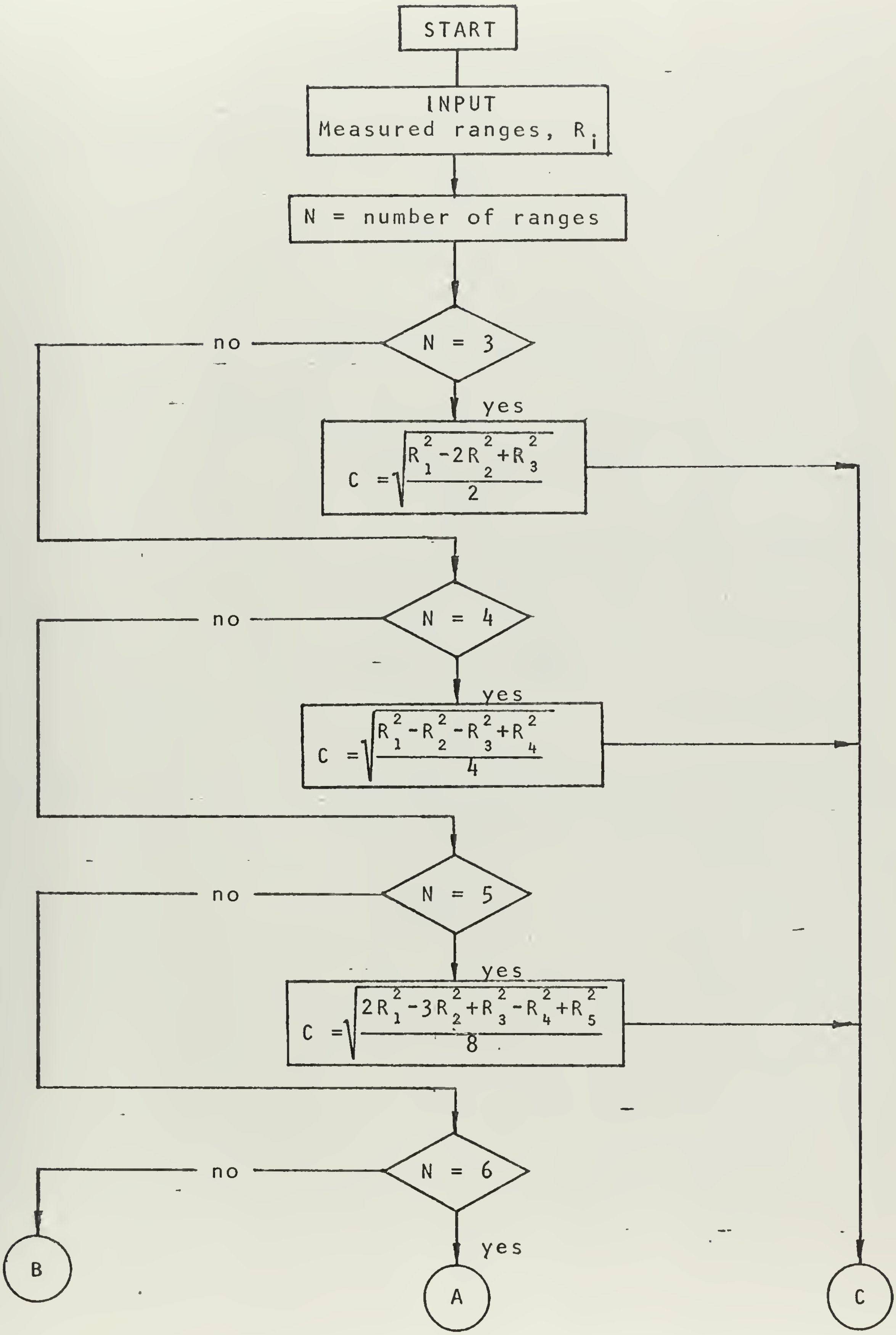
Calculate buoy coordinates X & Y. X and Y = polynomials in terms of X and Y. Loop generates iterative convergence process using previous values of X and Y to calculate new values

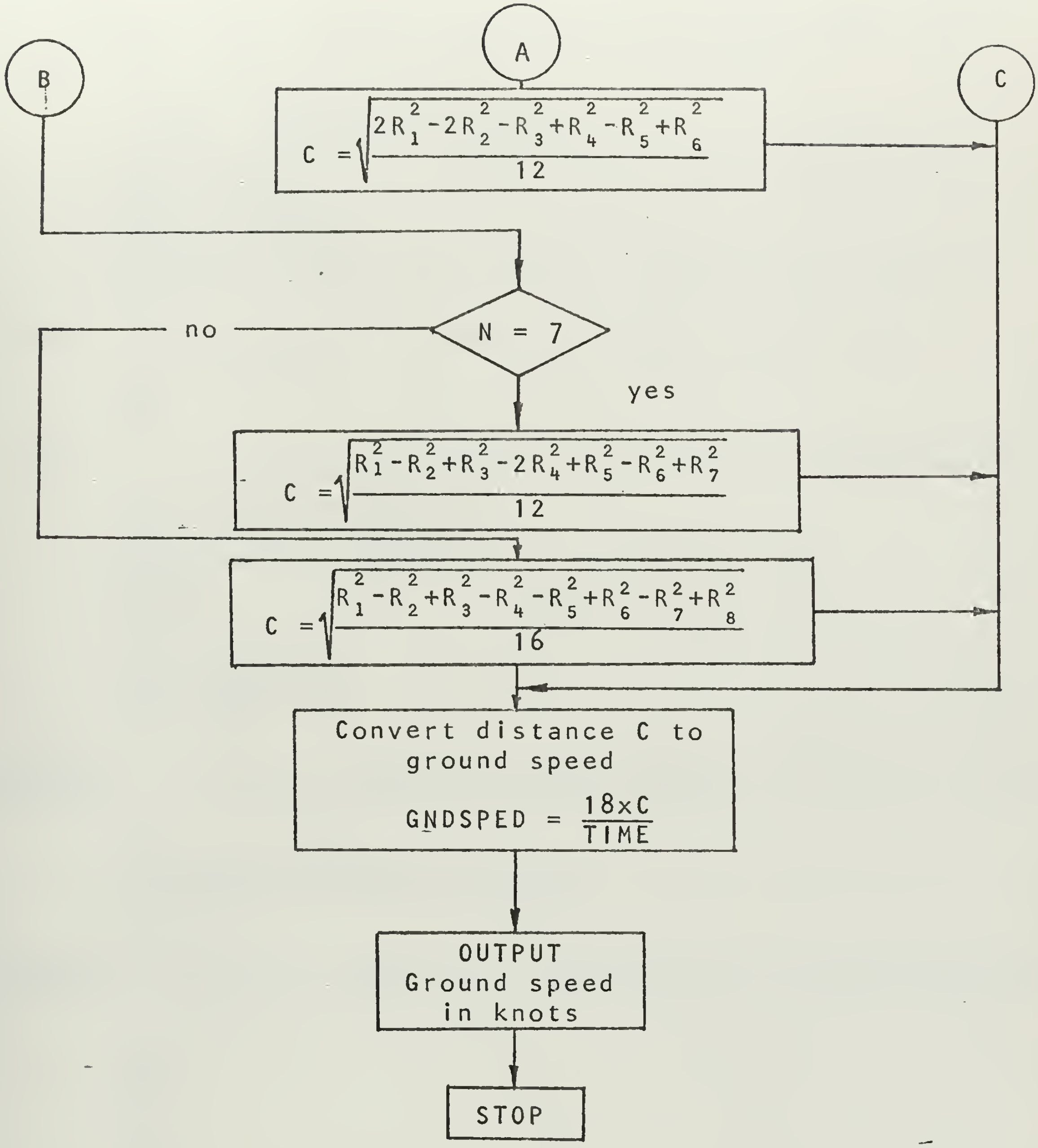
B

A


A
B
Establish error in each iteration
ERRORX = C-X
ERRORY = D-Y
is ERRORX ≤ TESTX
no
yes
is ERRORY ≤ TESTY
no
yes
OUTPUT
X,Y values of buoy coordinates and number of iterations required to converge to within acceptable values
STOP

GROUND SPEED DETERMINATION PROGRAM




A
B
C
$C = \sqrt{\frac{2R_1^2 - 2R_2^2 - R_3^2 + R_4^2 - R_5^2 + R_6^2}{12}}$
no
N = 7
yes
$C = \sqrt{\frac{R_1^2 - R_2^2 + R_3^2 - 2R_4^2 + R_5^2 - R_6^2 + R_7^2}{12}}$
$C = \sqrt{\frac{R_1^2 - R_2^2 + R_3^2 - R_4^2 - R_5^2 + R_6^2 - R_7^2 + R_8^2}{16}}$
Convert distance C to ground speed
$GNDSPED = \frac{18 \times C}{TIME}$
OUTPUT
Ground speed
in knots
STOP

```
SONOBUOY LOCATION PROGRAM. WITH MODIFICATIONS, TO BE USED
FOR ACTUAL DETERMINATION OF BUOY COORDINATES. INPUT VALUES
ARE ACTUAL RANGE MEASUREMENTS TAKEN, THE ASSOCIATED A/C
COORDINATES AT TIME RANGES TAKEN AND AN ESTIMATE OF THE REL-
ATIVE BUOY ANGLE WITH RESPECT TO A/C HEADING.

      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 DSQRT,DCOS,DSIN,DATAN,P(10),PP(10),Q(10),
     1QQ(10),S(10),SS(10),T(10),R(10),A(10),B(10),DABS
    1 FORMAT(F10.4)
    2 FORMAT(10F8.4)
    3 FORMAT(10F8.4)
    4 FORMAT(10F10.5)
    5 FORMAT('0'T7,'X',T17,'Y',T24,'ERRORX',T34,'ERRORY'/)
    6 FORMAT(' '4F10.4)

INPUT THE ESTIMATE OF THE RELATIVE ANGLE OF THE BUOY.

      READ(5,1)THETA
      N=3
      Z=N

INPUT THE A/C COORDINATES (A,B) AT EACH RANGE MEASUREMENT.

      READ(5,2)(A(I),I=1,N)
      READ(5,3)(B(I),I=1,N)
      READ(5,4)(R(I),I=1,N)
      ABAR=0.
      BBAR=0.
      RBAR=0.
      DO 50 I=1,N
      ABAR=ABAR+A(I)
      BBAR=BBAR+B(I)
      RBAR=RBAR+R(I)
   50 CONTINUE

CALCULATE INITIAL VALUES OF X AND Y (BUOY COORDINATES) USING
ESTIMATED VALUE OF THE RELATIVE BUOY ANGLE.

      RAD=57.29577951
      THETAR=THETA/RAD
      X=(RBAR*DCOS(THETAR)+ABAR)/Z
      Y=(RBAR*DSIN(THETAR)+BBAR)/Z
      WRITE(6,5)

COMMENCE ITERATIVE PROCESS FOR CONVERGENCE ON VALUES FOR THE
BUOY COORDINATES. NUMBER OF ITERATIONS MAY BE INPUT HERE.

      DO 100 J=1,15
      V1=0.
      V2=0.
      V3=0.
      V4=0.
      V5=0.
      V6=0.
      V7=0.
      DO 70 I=1,N
      P(I)=((X-A(I))**2+(Y-B(I))**2)**1.5
      PP(I)=((X-A(I))**2+(Y-B(I))**2)**.5
      Q(I)=(X-A(I))**2
      QQ(I)=X-A(I)
      S(I)=(Y-B(I))**2
      SS(I)=Y-B(I)
      T(I)=(X-A(I))*(Y-B(I))
      V1=V1+R(I)*S(I)/P(I)
      V2=V2+R(I)*T(I)/P(I)
      V3=V3+R(I)*Q(I)/P(I)
      V4=V4+R(I)*QQ(I)/PP(I)
      V5=V5+R(I)*SS(I)/PP(I)
      V6=V6+A(I)
      V7=V7+B(I)
   70 CONTINUE
```

```
      D1=1-V1/Z
      D2=V2/Z
      D3=D2
      D4=1-V3/Z
      D5=(Y*V2-X*V1+V4+V6)/Z
      D6=(X*V2-Y*V3+V5+V7)/Z
      X=(D5*D4-D3*D6)/(D1*D4-D3*D2)
      Y=(D1*D6-D5*D2)/(D1*D4-D3*D2)
  100 CONTINUE
C
C     WRITE OUT FINAL VALUES OF BUOY COORDINATES AT END OF ITER-
C     ATIVE PROCESS.
C
      WRITE(6,6)X,Y
      STOP
      END
```

```
SONOBUOY LOCATION SENSITIVITY ANALYSIS PROGRAM.  DATA OUTPUT
FROM PROGRAM DETERMINES THE RATE OF CONVERGENCE TO WITHIN
ACCEPTABLE VALUES OF THE SONOBUOY COORDINATES AS AFFECTED BY
VARIANCE OF THE FOLLOWING; NUMBER OF RANGE MEASUREMENTS USED
AND THE AVERAGE RANGE TO THE BUOY, RELATIVE ANGLE OF THE
SONOBUOY TO THE AIRCRAFT HEADING AND RELATIVE BEARING ESTI-
MATE.

      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 DSQRT,DCOS,DSIN,DATAN,P(10),PP(10),Q(10),
     1QQ(10),S(10),SS(10),T(10),R(10),A(10),B(10)
    1 FORMAT(3F10.4)
    2 FORMAT(10F8.4)
    3 FORMAT(10F8.4)
    5 FORMAT('0'////)
    6 FORMAT('0'T5,'THETA',T17,'X',T27,'Y',T37,'C',T47,'D'/)
    7 FORMAT('0'5F10.4//)
    8 FORMAT(' '2F10.4,I10)

ESTABLISH THE NUMBER OF RANGE MEASUREMENTS TAKEN.

      N=7
      Z=N

READ INPUT VALUES OF A/C COORDINATES FOR EACH OF THE RANGE
MEASUREMENTS.

      READ(5,2)(A(I),I=1,N)
      READ(5,3)(B(I),I=1,N)
      GO TO 400
  200 WRITE(6,5)

READ INPUT VALUES OF THE RELATIVE BEARING ESTIMATE AND THE
ACTUAL BUOY COORDINATES.

  400 READ(5,1)THETA,C,D
      IF(C.EQ.1000)GO TO 300
      ABAR=0
      BBAR=0

DETERMINE ACTUAL RANGES FROM A/C TO BUOY.

      DO 50 I=1,N
      R(I)=DSQRT((C-A(I))**2+(D-B(I))**2)
      ABAR=ABAR+A(I)
      BBAR=BBAR+B(I)
   50 CONTINUE

CALCULATE ACCEPTABLE ERROR (TEST) AS .5% OF RANGE TO BUOY.

      TESTX=R(2)/200.
      TESTY=TESTX
      RAD=57.29577951
      RBAR=0
      DO 60 I=1,N
      RBAR=RBAR+R(I)
   60 CONTINUE

DETERMINE INITIAL VALUES OF BUOY COORDINATES USING ESTIMATED
VALUE OF RELATIVE BEARING.

      THETAR=THETA/RAD
      X=(RBAR*DCOS(THETAR)+ABAR)/Z
      Y=(RBAR*DSIN(THETAR)+BBAR)/Z
      WRITE(6,6)
      WRITE(6,7)THETA,X,Y,C,D
```

COMMENCE ITERATIVE PROCESS TO CONVERGE ON ACTUAL VALUES OF BUOY COORDINATES.

```
      DO 100 J=1,20
      V1=0
      V2=0
      V3=0
      V4=0
      V5=0
      V6=0
      V7=0
```

CALCULATE COEFFICIENT MATRIX AND CONSTANTS FOR SYSTEM OF NORMAL EQUATIONS USED IN THE ITERATIVE PROCESS.

```
      DO 70 I=1,N
      P(I)=((X-A(I))**2+(Y-B(I))**2)**1.5
      PP(I)=((X-A(I))**2+(Y-B(I))**2)**.5
      Q(I)=(X-A(I))**2
      QQ(I)=X-A(I)
      S(I)=(Y-B(I))**2
      SS(I)=Y-B(I)
      T(I)=(X-A(I))*(Y-B(I))
      V1=V1+R(I)*S(I)/P(I)
      V2=V2+R(I)*T(I)/P(I)
      V3=V3+R(I)*Q(I)/P(I)
      V4=V4+R(I)*QQ(I)/PP(I)
      V5=V5+R(I)*SS(I)/PP(I)
      V6=V6+A(I)
      V7=V7+B(I)
   70 CONTINUE
      D1= 1-V1/Z
      D2=V2/Z
      D3=D2
      D4=1-V3/Z
      D5=(Y*V2-X*V1+V4+V6)/Z
      D6=(X*V2-Y*V3+V5+V7)/Z
```

USE KRAMER'S RULE WITH RESPECT TO THE COEFFICIENTS CALCULATED ABOVE TO DETERMINE VALUES OF BUOY COORDINATES.

```
      X=(D5*D4-D3*D6)/(D1*D4-D3*D2)
      Y=(D1*D6-D5*D2)/(D1*D4-D3*D2)
      WRITE(6,8)X,Y,J
```

IF ERROR IS WITHIN ACCEPTABLE VALUE, STOP ITERATIVE PROCESS AND NOTE THE NUMBER OF ITERATIONS REQUIRED TO CONVERGE AND THE FINAL BUOY COORDINATES.

```
      TESTXX=DABS(C-X)
      TESTYY=DABS(D-Y)
      IF(TESTXX.GT.TESTX)GO TO 100
      IF(TESTYY.GT.TESTY)GO TO 100
      GO TO 200
  100 CONTINUE
      GO TO 200
  300 STOP
      END
```

GROUND SPEED DETERMINATION PROGRAM. FOR USE IN THE ACTUAL DETERMINATION OF GROUND SPEED OF THE AIRCRAFT. INPUTS SUPPLIED ARE TIME BETWEEN RANGE MEASUREMENTS AND THE ACTUAL RANGES MEASURED.

```
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 DSQRT,R(10)
    1 FORMAT(10F8.4)
    2 FORMAT('0'F12.4)
```

ESTABLISH NUMBER OF RANGES TAKEN AND TIME BETWEEN RANGES IN SECONDS.

```
      N=3
      Z=N
      TIME=3.
```

INPUT MEASURED RANGES.

```
      READ(5,1)(R(I),I=1,N)
      IF(N.GT.3)GO TO 8
      GS=DSQRT((R(1)**2-2.*R(2)**2+R(3)**2)/2.)
      GO TO 20
    8 IF(N.GT.4)GO TO 9
      GS=DSQRT((R(1)**2-R(2)**2-R(3)**2+R(4)**2)/4.)
      GO TO 20
    9 IF(N.GT.5)GO TO 10
      GS=DSQRT((2.*R(1)**2-3.*R(2)**2+R(3)**2-R(4)**2+
     1R(5)**2)/8.)
      GO TO 20
   10 IF(N.GT.6)GO TO 11
      GS=DSQRT((2.*R(1)**2-2.*R(2)**2-R(3)**2+R(4)**2
     1-R(5)**2+R(6)**2)/12.)
      GO TO 20
   11 IF(N.GT.7)GO TO 12
      GS=DSQRT((R(1)**2-R(2)**2+R(3)**2-2.*R(4)**2+R(5)**2
     1-R(6)**2+R(7)**2)/12.)
      GO TO 20
   12 GS=DSQRT((R(1)**2-R(2)**2+R(3)**2-R(4)**2-R(5)**2
     1+R(6)**2-R(7)**2+R(8)**2)/16.)
   20 GNDSPD=GS*18./TIME
      WRITE(6,2)GNDSPD
      STOP
      END
```

```
GROUND SPEED DETERMINATION SENSITIVITY ANALYSIS PROGRAM.
DATA OUTPUT RELATES TO GROUND SPEED ERROR AS AFFECTED BY
VARIANCE OF RANGE ERRORS, RELATIVE BUOY ANGLE, RANGE TO
BUOY, NUMBER OF RANGES MEASURED AND TIME BETWEEN RANGE
MEASUREMENTS.

      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 DSQRT,DABS,R(10),A(10),B(10),ERR(10)
    1 FORMAT(2F10.4)
    2 FORMAT(8F10.4)
    5 FORMAT('0'T1,'GROUNDSPEED'T14,'ERROR')
    6 FORMAT(' '2F10.4)

ESTABLISH ACTUAL DISTANCE FLOWN IN NAUTICAL MILES BETWEEN
RANGE MEASUREMENTS, TIME BETWEEN RANGE MEASUREMENTS AND
ACTUAL GROUND SPEED TO BE USED AS A TEST FOR DATA GROUND-
SPEED.

      E=.25
      TIME=4.5
      TEST=E*3600./TIME
      N=6
      Z=N

INPUT ACTUAL BUOY COORDINATES TO SIMULATE VARYING BUOY
ANGLES AND RANGES RELATIVE TO THE AIRCRAFT.

      READ(5,1)C,D

INPUT AIRCRAFT COORDINATES AND RANGE ERROR APPLIED TO
ACTUAL RANGE TO SIMULATE MEASURED RANGE WITH ERROR.

      READ(5,2)(A(I),I=1,N)
      READ(5,2)(B(I),I=1,N)
      WRITE(6,5)
      DO 100 J=1,70
      READ(5,2)(ERR(I),I=1,N)

CALCULATE MEASURED RANGE AS DIFFERING FROM ACTUAL RANGE
BY ERROR APPLIED.
      DO 200 I=1,N
      R(I)=DSQRT((C-A(I))**2+(D-B(I))**2)
      R(I)=R(I)+ERR(I)
  200 CONTINUE

CALCULATE VALUE OF GROUND SPEED FROM INPUT DATA.

      IF(N.GT.3)GO TO 30
      GS=DSQRT((R(1)**2-2.*R(2)**2+R(3)**2)/2.)
      GO TO 50
   30 IF(N.GT.4)GO TO 31
      GS=DSQRT((R(1)**2-R(2)**2-R(3)**2+R(4)**2)/4.)
      GO TO 50
   31 IF(N.GT.5)GO TO 32
      GS=DSQRT((2.*R(1)**2-3.*R(2)**2+R(3)**2-R(4)**2
     1+R(5)**2)/8.)
      GO TO 50
   32 IF(N.GT.6)GO TO 33
      GS=DSQRT((2.*R(1)**2-2.*R(2)**2-R(3)**2+R(4)**2
     1-R(5)**2+R(6)**2)/12.)
      GO TO 50
   33 IF(N.GT.7)GO TO 34
      GS=DSQRT((R(1)**2-R(2)**2+R(3)**2-2.*R(4)**2+R(5)**2
     1-R(6)**2+R(7)**2)/12.)
      GO TO 50
   34 GS=DSQRT((R(1)**2-R(2)**2+R(3)**2-R(4)**2-R(5)**2
     1+R(6)**2-R(7)**2+R(8)**2)/16.)
   50 GNDSPD=GS*3600./TIME
```

```
TEST CALCULATED GROUND SPEED AGAINST ACTUAL GROUND SPEED TO
DETERMINE ERROR.

      ERROR=DABS(TEST-GNDSPD)
  100 WRITE(6,6)GNDSPD,ERROR
      STOP
      END
```

LIST OF REFERENCES

1. Ellis, J. R., Design and Evaluation of a Sonobuoy Ranging System, N. S. Thesis, Naval Postgraduate School, Monterey, California, June 1970.

2. Grant, G. M., Preliminary Investigation of a Proposed Sonobuoy Ranging System, M. S. Thesis, Naval Postgraduate School, Monterey, California, June 1969.

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13. ABSTRACT

In airborne Anti-Submarine Warfare operations there is a critical requirement for maintaining an accurate relative plot of the sonobuoys with respect to the aircraft. This study proposed a method for locating sonobuoys in a pattern using aircraft-to-buoy slant range information. The method did not use triangulation procedures and attempted to minimize the restrictions placed on the aircraft. The study showed the feasibility of the proposed methodology and the approximate errors to be encountered.

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KEY WORDS	LINK A		LINK B		LINK C	
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Position location						
Anti-Submarine Warfare						
Rante-only positioning						
Ground speed determination						

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Sonobuoy location.